Ockham's Razor, Stability, and Truth Conduciveness

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Supported by a grant from the John Templeton Foundation.

Manuscript: http://www.andrew.cmu.edu/user/kk3n/simplicity/bulletin-12.pdf

2nd November 2014

Belief Revision Theory

Two Overlooked Constraints

1. Inductive belief revision in science is rational.
2. Rationality should be truth conducive.

Scientific Example (Morrison 2000)

Possible Conclusions

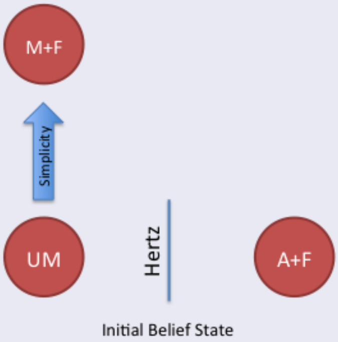
1. Ampere + Fresnel: no *EMR*.
2. Unified Maxwell: *EMR* exists; $LR = EMR$.
3. Maxwell + Fresnel: *EMR* exists; $LR \neq EMR$.

Possible Information States

1. Hertz: can decide *EMR* existence.
2. Fizeau: can refute but not verify $LR = EMR$.

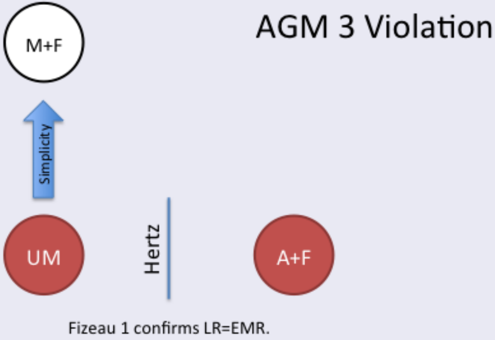
Scientific Example

Stage 0



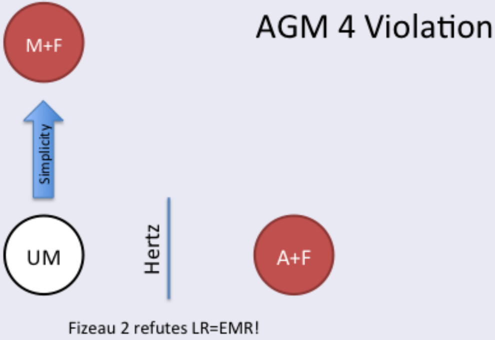
Scientific Example

Stage 1



Scientific Example

Stage 2



Scientific example

Two Morals

1. The AGM postulates do not govern inductive rationality.
2. The right postulates depend on empirical simplicity.

Ockham's Razor

What it is

- A bias toward simpler theories compatible with experience.
- An essential principle of inductive inference.

Ockham's Razor

Three Questions for Inductive Belief Revision Theory

I. What is empirical simplicity?
II. Given simplicity, what is Ockham's razor?
III. How does Ockham's razor help you find the truth better than alternative methods?

I. What is Empirical Simplicity?

Standard Accounts

- uniformity of nature (expressed in grue/bleen?)
- entrenchment (Maxwell had past success with electromagnetic fields?)
- generation by a brief computer program (in Turing machines or Java?)

Our Account

- Empirical simplicity reflects iterated problems of induction in an empirical problem context.

I. What is Empirical Simplicity?

Empirical Problem Contexts

- Inquiry is guided by an empirical problem context $\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$.
- W is the set of possible worlds.
- $\mathcal{I}$ is a set of possible information states such that:
  - the information states cover W;
  - every pair of true information states is entailed by a true information state.
- $\mathcal{Q}$ is a question that partitions W into countably many answers.

I. What is Empirical Simplicity?

Verifiability and Topology

- The closure $\mathcal{I}^*$ of $\mathcal{I}$ under union is a topology on W.
- The open sets are propositions verifiable by information.
- The closed sets are propositions refutable by information.
- The closure $\overline{A}$ of A consists of the worlds in which A is never refuted.

Some Related Approaches

- Kelly (1996)
- Luo and Schulte (2006)
- deBrecht and Yamamoto (2008)
- Baltag, Smets, and Gierasimczuk (2014)

I. What is Empirical Simplicity?

The Empirical Simplicity Order

- $A \preceq B$ iff $A \subseteq \overline{B}$
- iff A entails that B will never be refuted
- iff the problem of induction obtains from A to B
- iff A is as falsifiable as B (Popper).

I. What is Empirical Simplicity?

Two Wrinkles

1. $(\mathcal{Q}, \preceq)$ can have cycles.
2. $(\mathcal{Q}, \preceq)$ can be unstable under restriction by new information.

I. What is Empirical Simplicity?

Solution

- Substitute a better question $\mathcal{S}$ for the original question $\mathcal{Q}$.
- Call $\mathcal{S}$ a simplicity concept for $\mathfrak{P}$ and call answers to $\mathcal{S}$ simplicity degrees.
- $\mathcal{S}$ is related to $\mathfrak{P}$ by three axioms.

I. What is Empirical Simplicity?

Axiom 1. Local Closure

- A is locally closed for $\mathfrak{P}$ iff $A = B \setminus C$, where B, C are open (verifiable)
- iff A implies that A will be refutable.
- Then it is safe to infer A if B is verified until C is verified.
- **Proposition**. If each simplicity degree in $\mathcal{S}$ is locally closed, then $(\mathcal{S}, \preceq)$ is anti-symmetric.

I. What is Empirical Simplicity?

Axiom 2. Homogeneity

- $\mathcal{S}$ is homogeneous for $\mathfrak{P}$ iff

$$\{w\} \preceq C \quad \Rightarrow \quad \mathcal{S}_w \preceq C,$$

  for all $w \in W$ and $C \in \mathcal{S}$.

- **Proposition**. The simplicity relation $\preceq$ is stable under restriction by information iff $\mathcal{S}$ is homogeneous for $\mathfrak{P}$.

I. What is Empirical Simplicity?

Axiom 3. Decides the Original Problem

- $\mathcal{S}$ decides $\mathfrak{P}$ iff each answer to $\mathcal{Q}$ is open (verifiable) in the information topology restricted to an arbitrary simplicity degree in $\mathcal{S}$.

II. What is Ockham's Razor?

Ockham's Vertical Razor

- Your belief state should be closed downward in $\preceq$.

Ockham's Horizontal Razor

Your belief state should be co-initial in $\preceq$.

III. How Does Ockham's Razor Help You Find the Truth?

Bayesian answer

- Simpler worlds are more probable, so Ockham's Razor is probably right.
- Converges to the truth in the long run, but so do infinitely many other methods.

Frequentist answer

- Estimates based on simpler models have lower variance.
- Doesn't converge to the true model at all (AIC).

III. How Does Ockham's Razor Help You Find the Truth?

Ancient Hint (Katha Upanishad, Müller Translation)

- Fools dwelling in darkness, wise in their own conceit, and puffed up with vain knowledge
  - go round and round
  - staggering to and fro,

  like blind men led by the blind.

III. How Does Ockham's Razor Help You Find the Truth?

Staggering To and Fro = Doxastic Reversals

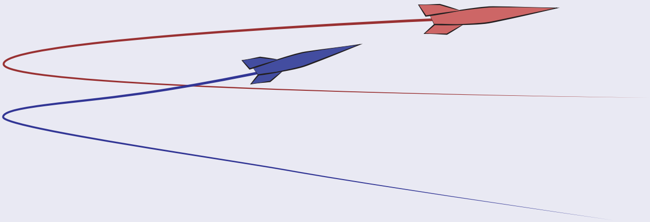
1. Believe A;
2. believe B inconsistent with A.

Going Round and Round = Doxastic Cycles

1. Believe A;
2. believe B inconsistent with A;
3. believe C that entails A.

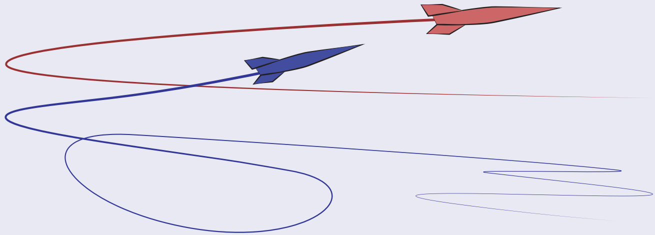
III. How Does Ockham's Razor Help You Find the Truth?

Conducive Pursuit



III. How Does Ockham's Razor Help You Find the Truth?

Non-conducive Pursuit



III. How Does Ockham's Razor Help You Find the Truth?

Direct Comparison of Reversal and Cycle Sequences

- $\sigma \leq \tau$ iff there exists sub-sequence τ' of τ such that $\sigma_i \subseteq \tau'_i$, for all $i \leq$ length of σ.

III. How Does Ockham's Razor Help You Find the Truth?

Worst-case Comparisons over Simplicity Degrees

- Let $E \in \mathcal{I}$.
- $\lambda \leq_E^{\text{rev}} \lambda'$ iff for each reversal sequence σ generated by λ, in world $w \in E$, there exists reversal sequence τ produced by λ' in world $v \in C \cap E$ such that $\sigma \leq \tau$.
- Similarly for cycles.

III. How Does Ockham's Razor Help You Find the Truth?

Optimality

- λ is retraction optimal in $\mathfrak{S}$ iff
  1. λ solves $\mathfrak{S}$ in the limit;
  2. $\lambda \leq_E^{\text{rev}} \lambda'$, for all $E \in \mathcal{I}$ and for all λ' that solve $\mathfrak{S}$ in the limit.
- Similarly for cycle-optimality.

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III. How Does Ockham's Razor Help You Find the Truth?

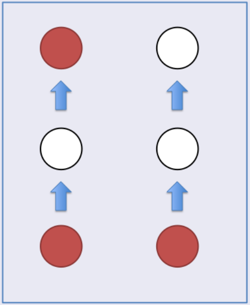
Sub-optimality

- λ is retraction sub-optimal in $\mathfrak{S}$ iff
  1. λ does not solve $\mathfrak{S}$ in the limit or
  2. $\lambda' < \text{rev}_E \lambda'$, for some $E \in \mathcal{I}$ and for some λ' that solves $\mathfrak{S}$ in the limit.
- Similarly for cycle-optimality.

Justification of Ockham's Razor

Vertical Razor

- Necessary for cycle optimality.
- Necessary for avoidance of cycle sub-optimality.



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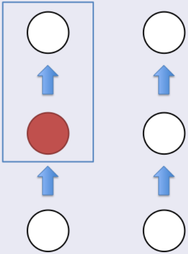

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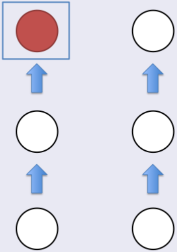

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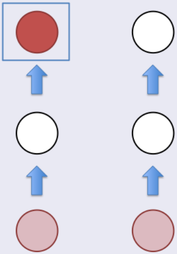
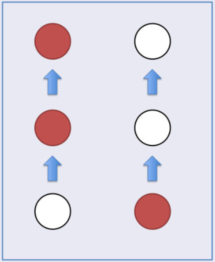


Justification of Ockham's Razor

Horizontal Razor

Assume that lower bounds on empirical complexity are verifiable. Then:

- Necessary and sufficient for reversal optimality.
- Necessary and sufficient for avoidance of reversal sub-optimality.



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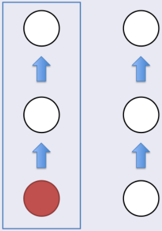

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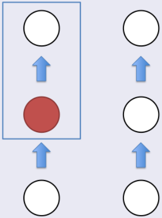

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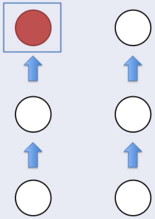

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Thank you!

Supported by a grant from the John Templeton Foundation.

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