The cost of consistency
Contraction and the loss of true belief

Ted Shear [1]   Konstantin Genin [2]

[1]University of California, Davis
ebshear@ucdavis.edu

[2]Carnegie Mellon University
kgenin@andrew.cmu.edu

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Outline

The main result and idealizations

Main result

There are inconsistent corpora that give agents no choice but to drop a true belief if they are to resolve their inconsistency.

Idealizations on agents:

I. Corpora of beliefs are *not* closed under consequence.

II. Agents have awareness of salient events, perfect recall of those events, and an awareness of that perfect recall.

III. Agents contract their corpora by some non-empty set of beliefs only if they have inconsistent corpora.

IV. There are *no other restrictions* on how agents can or should contract.

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The paradoxical scenario

- A teacher announces to her students at the end of the day on a Wednesday:

  'There will be an exam sometime this week, but you will not be able to deduce from your beliefs on the morning of the day of the exam that the exam will be that day.'

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Formalising the announcement

- Assume the following readings for the two atomic sentences:

$$p_1 \quad : \quad \text{'there is an exam at } t_1\text{'}$$
$$p_2 \quad : \quad \text{'there is an exam at } t_2\text{'}$$

- Where α is some agent, φ is some sentence, and t_n is some time, let:

$$\mathrm{Prv}_n\varphi \quad : \quad \text{'}\varphi \text{ is provable from } \alpha\text{'s corpus of beliefs on the morning of } t_n\text{'}$$

- Let the teacher's announcement, A, be the following sentence:

$$A := (p_1 \veebar p_2) \wedge (p_1 \supset \neg\mathrm{Prv}_1 p_1) \wedge (p_2 \supset \neg\mathrm{Prv}_2 p_2)$$

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- Where α is an agent and φ is some sentence, let:

$$B\varphi \quad : \quad \text{`}\alpha \text{ believes that } \varphi\text{'}$$
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Alice's corpus of beliefs

- Let Alice's corpus of beliefs at t_0 be the set:

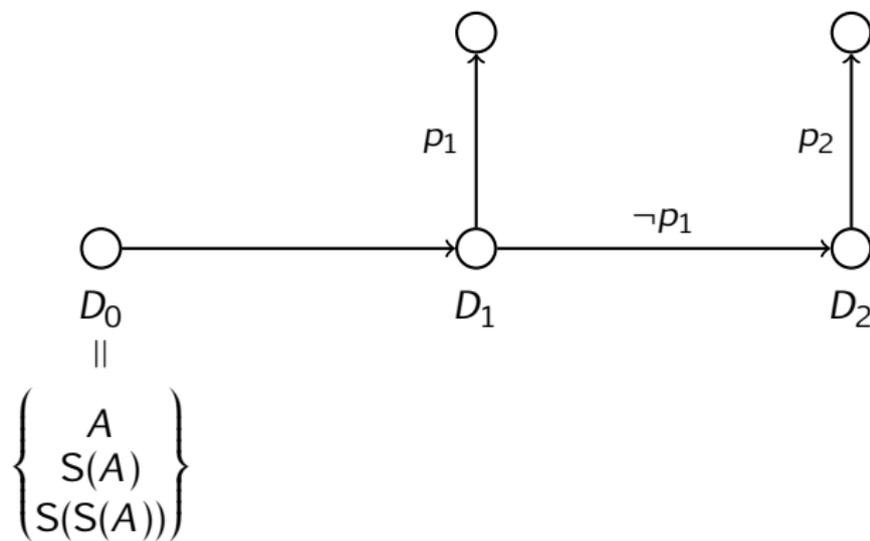$$D_0 = \{A, S(A), S(S(A))\}$$

- We will see that $D_0 \vdash \bot$.

Alice's corpus of beliefs

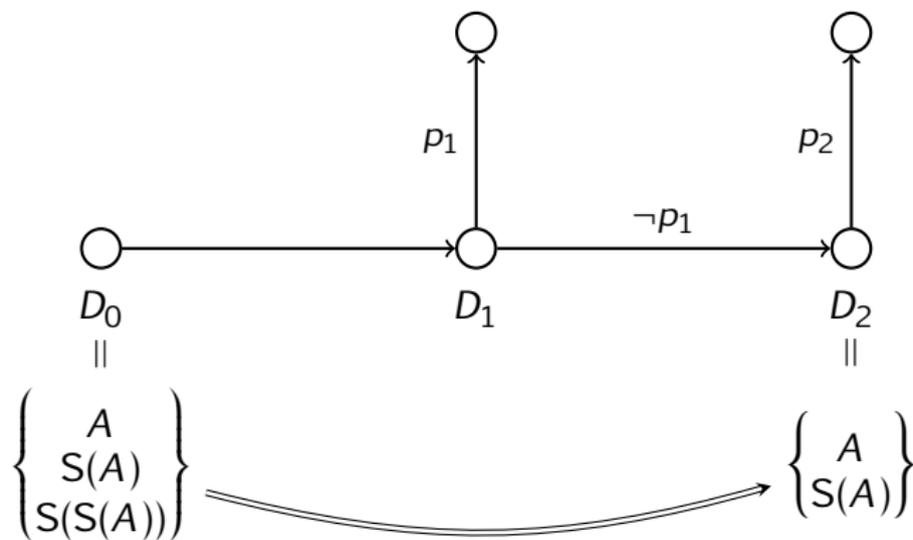- Let Alice's corpus of beliefs at t_0 be the set:

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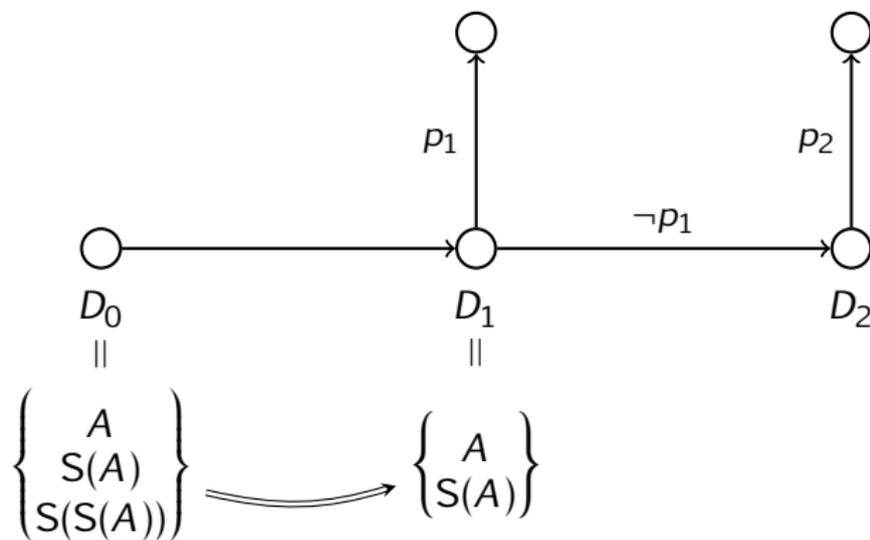
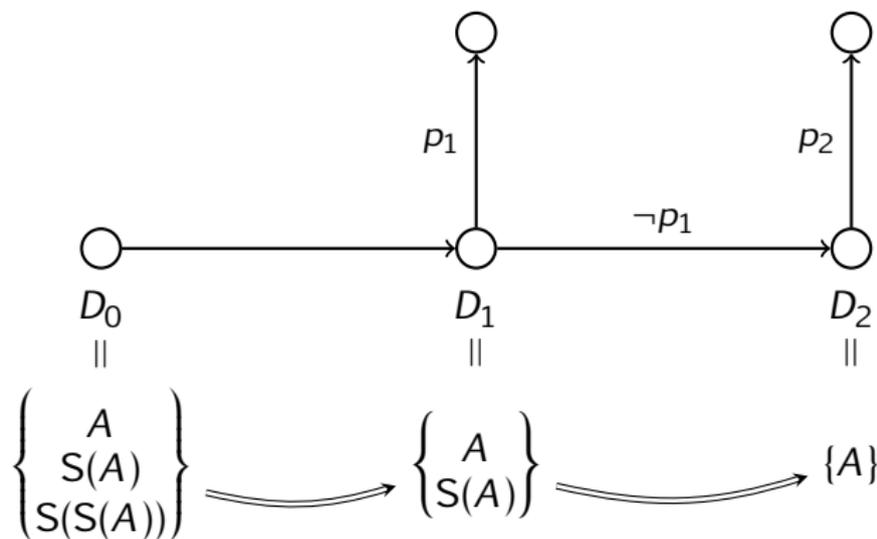# Demonstrating the inconsistency

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Options for contraction

- What are her options for contraction on Wednesday?

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  (ii) Contract by some set containing $S(A)$, but not containing A:

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  (iii) Contract by the singleton $\{S(S(A))\}$.

- We will prove by cases that as long as there is any exam at all, Alice will lose a true belief no matter how she contracts.

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Option (i): Contract by a set containing *A*

- This is the contraction in the standard Surprise Examination Paradox. When Alice contracts by A, she falsifies $S(A)$. So, her reasoning depended on the false belief that she is stable and, thus, the paradox is resolved.[1]

- If Alice contracts by any set containing A, then she will not be able to prove either p_1 or p_2.

- It follows that whenever the exam occurs A will be true and so Alice will have contracted by a true belief.

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Option (ii): Contract by a set containing S(A), but not A

- Alice is now consistent, so she can contract no further. Therefore GBA, the consequent of $S(A)$, is satisfied and she has contracted by a true belief.
- When Alice contracts by S(A), she falsifies S(S(A)).

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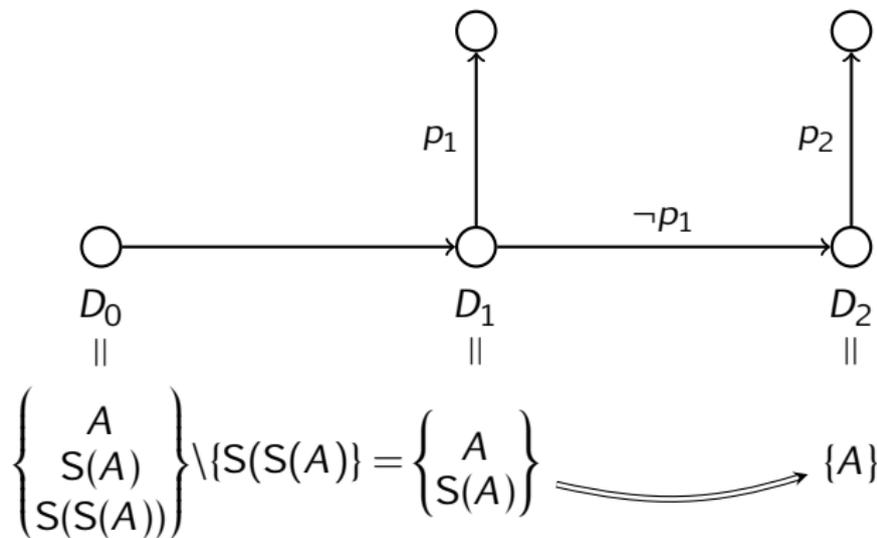
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Option (iii): Contract by {S(S(A))}

- This case is a little bit trickier. When Alice contracts by {S(S(A))}, she arrives on Thursday morning with the corpus $D_1 = \{A, S(A)\}$.

$$D_0 \quad\quad\quad\quad D_1 \quad\quad\quad\quad D_2$$
$$\| \quad\quad\quad\quad\quad \| \quad\quad\quad\quad\quad \|$$

$$\left\{ \begin{array}{c} A \\ S(A) \\ S(S(A)) \end{array} \right\} \setminus \{S(S(A))\} = \left\{ \begin{array}{c} A \\ S(A) \end{array} \right\} \Longrightarrow \{A\}$$

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  - (iii) Contract by the singleton $\{S(S(A))\}$.
    - (iii-a) Contract by $\{A\}$;
    - (iii-b) Contract by a set containing $S(A)$: $\{S(A)\}$, or $\{A, S(A)\}$.

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- Under this strategy, Alice retains belief in S(A) and so the consequent of S(S(A)) is satisfied and her first contraction was by a true belief.

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Option (iii-b): Contract by {S(S(A))}, then by a set containing S(A)

- If the exam is on Thursday, then as we have seen A will be false. This means that S(A) will be true and she will have contracted by a true belief.
- If the exam is on Friday and she contracted by {S(A)}, then again A will be false and S(A) will be true.
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    - (iii) Contract by the singleton $\{S(S(A))\}$.
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Summary of the results of contraction

- As long as there is any exam at all, Alice will lose a true belief no matter how she contracts.

Option	First contraction	Second contraction	A	$S(A)$	$S(S(A))$
[i]	$\setminus\{X : A \in X\}$	—	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$
[ii]	$\setminus\{X : S(A) \in X, A \notin X\}$	—	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$
[iii-a]	$\setminus\{S(S(A))\}$	$\setminus\{A\}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$
[iii-b]	$\setminus\{S(S(A))\}$	$\setminus\{X : S(A) \in X\}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$

Table 1: If the exam is on Thursday

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[iii-a]	$\setminus\{S(S(A))\}$	$\setminus\{A\}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$
[iii-b]	$\setminus\{S(S(A))\}$	$\setminus\{S(A)\}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$
[iii-b]	$\setminus\{S(S(A))\}$	$\setminus\{A, S(A)\}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$

Table 2: If the exam is on Friday

How *should* Alice contract?

- What considerations might be relevant for assessing Alice's decision?

Some principles for epistemic decision theory

CONSERVATIVITY: Agents ought to minimally mutilate their corpus.

ENTRENCHMENT: Agents ought to drop least entrenched beliefs first.

CONSISTENCY: Agents' contractions ought to establish consistency.

TRUTH-SEEKING: Agents ought not to contract in ways that necessarily leave them with some false beliefs.

TRUTH-MAINTENANCE: Agents ought to drop only false beliefs.

How *should* Alice contract?

First contraction	Second contraction
$\backslash\{A, S(A), S(S(A))\}$	
$\backslash\{A\}$	
$\backslash\{A, S(A)\}$	
$\backslash\{A, S(S(A))\}$	
$\backslash\{S(A), S(S(A))\}$	
$\backslash\{S(S(A))\}$	$\backslash\{S(A)\}$
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Contractions admissible under TRUTH-SEEKING.

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How *should* Alice contract?

- Alice believes *A* on the testimony of the teacher. Presumably, the teacher counts as an epistemic authority.
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How *should* Alice contract?

- We have seen that if there is any exam at all, TRUTH-MAINTENANCE eliminates all contraction strategies.
- However, "no exam" is a live epistemic possibility for Alice.
- Alice can reestablish consistency *without* losing a true belief iff there is no exam and she contracts by only *A*.

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- Which behavior we recover depends on whether we prioritize ENTRENCHMENT over TRUTH-MAINTENANCE!

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Conclusion

- A close examination of the possible ways that Alice might contract in the Surprise Examination Paradox shows that if she wishes to resolve her inconsistency, she must give up a true belief.

- It seemed initially plausible that if an agent has an inconsistent corpus of beliefs, then she can somehow resolve the inconsistency in a way that does not require her to give up any true beliefs. We have provided a case in which precisely this can happen.

- Additionally, we have considered some plausible principles of epistemic decision theory and used them to evaluate Alice's options for contraction. We tentatively concluded that Alice ought to contract by $\{S(A), S(S(A))\}$.

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Thank you!

Ted Shear (University of California, Davis)
ebshear@ucdavis.edu

Konstantin Genin (Carnegie Mellon University)
kgenin@andrew.cmu.edu

Here's another demonstration

[DESCARTES] Let Descartes' corpus be $\{d, \neg d\}$, where:

$d :$ 'It will rain the day after my next contraction'.

Moreover, let an adversarial demon make it rain iff Descartes contracts by any set that contains d.

Another one!

[WILT] Let Wilt's corpus be $\{w_1, (Bw_1 \cup w_2), w_1 \equiv (\neg Bw_1 \cup w_2)\}$, where:

$$w_1: \quad \text{'I will make my next free-throw'};$$
$$w_2: \quad \text{'I attempt a free-throw'}.$$

Additionally, let U be the until operator of linear tense logic read as:

$$(\varphi \cup \psi): \quad \text{'}\varphi \text{ holds at least until } \psi\text{'}.$$

Moreover, let Wilt's psychological profile be such that he will make his free-throw iff it's not the case that he believes that he will make his free-throw until the time at which he attempts it.

And another case

[Cassie] Let Cassie's corpus be $\{\lambda, \neg\lambda\}$, where:

$\lambda :$ 'My next contraction will be by a set of sentences that entail λ'.

One final case

[VADER] Let Vader's corpus be $\{v_n : n \in \mathbb{N} \cup \{0\}\}$, where:

v_0 : 'I will contract and my next contraction will be by a finite set';

v_1 : 'My next contraction will not be by a subset of $\{v_0\}$';

v_2 : 'My next contraction will not be by a subset of $\{v_0, v_1\}$';

$\vdots$ $\vdots$

v_n : 'My next contraction will not be by a subset of $\bigcup\limits_{0 \leq i < n} \{v_i\}$'.