The Topology of Statistical Verifiability

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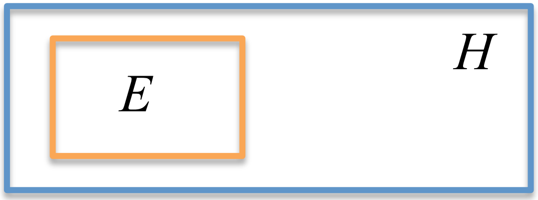
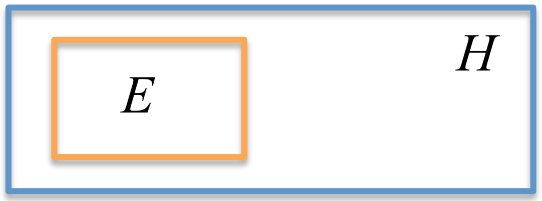
A Worry

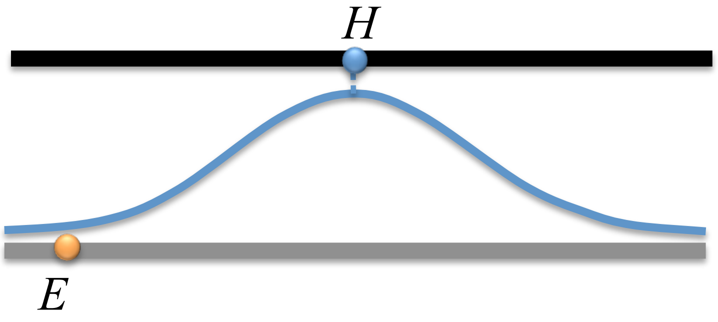
- Propositional information refutes logically incompatible possibilities.



A Worry

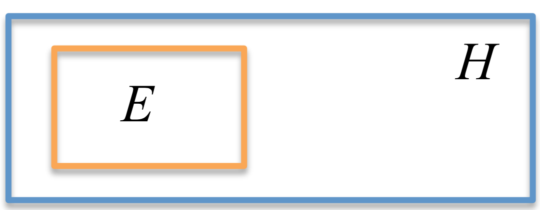
- Propositional information refutes logically incompatible possibilities.
- Typically, statistical samples are logically compatible with every possibility.

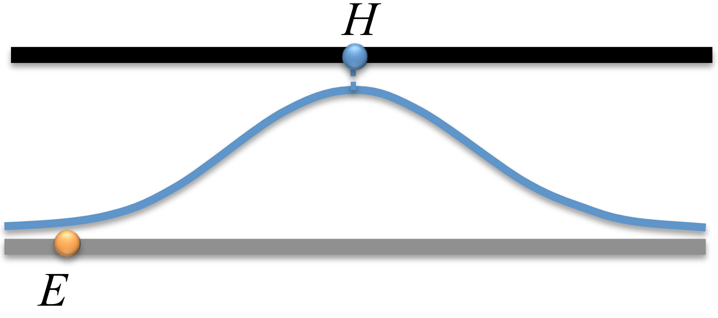




Response

Don't worry!

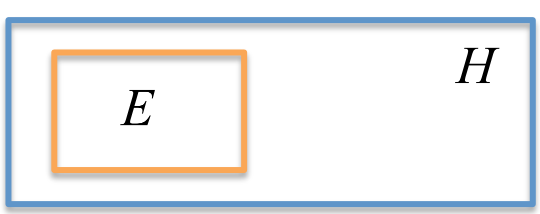


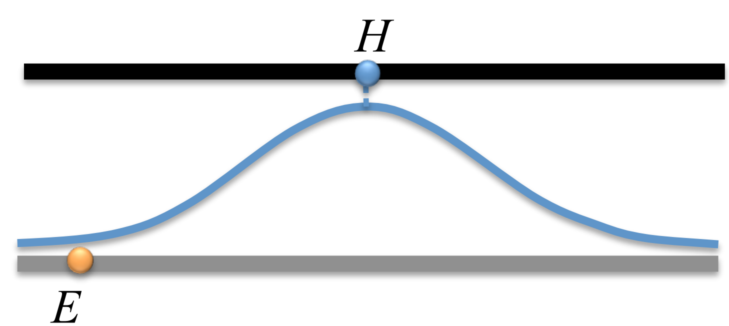


Response

Don't worry!

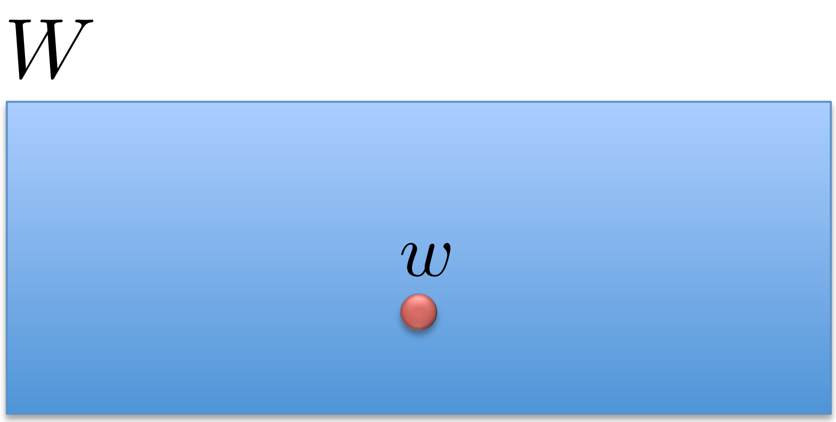
Common **topological** structure





TOPOLOGY AND LOGICAL VERIFIABILITY

Possible Worlds

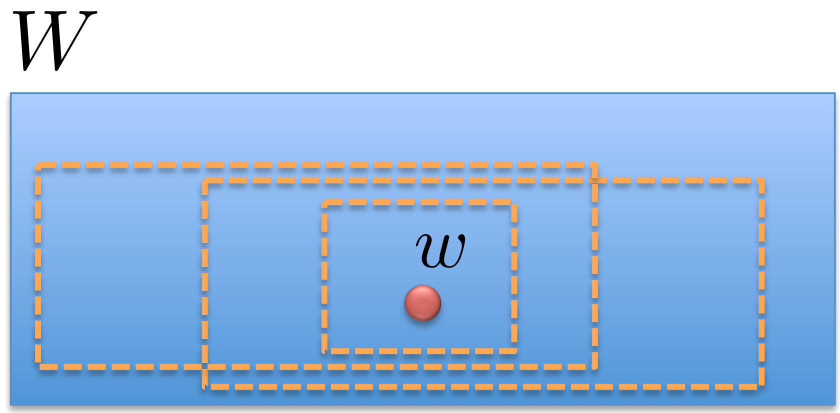


Propositional Information State

The logically strongest proposition you are informed of.

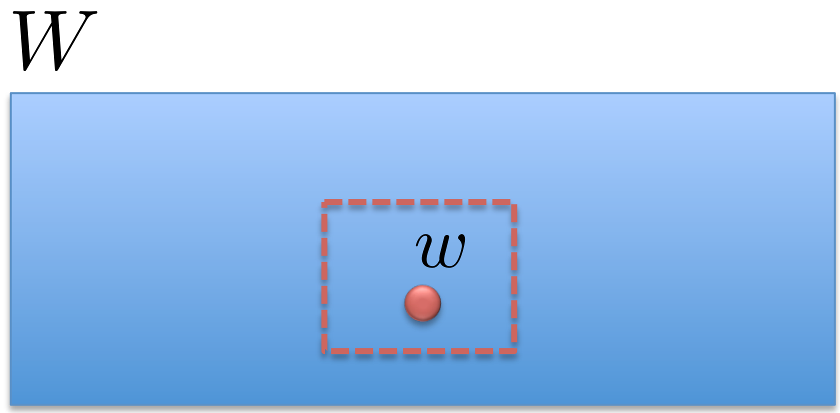
Information States

$$\mathcal{I} = \text{the set of all information states.}$$
$$\mathcal{I}(w) = \text{the set of all information states true in } w.$$



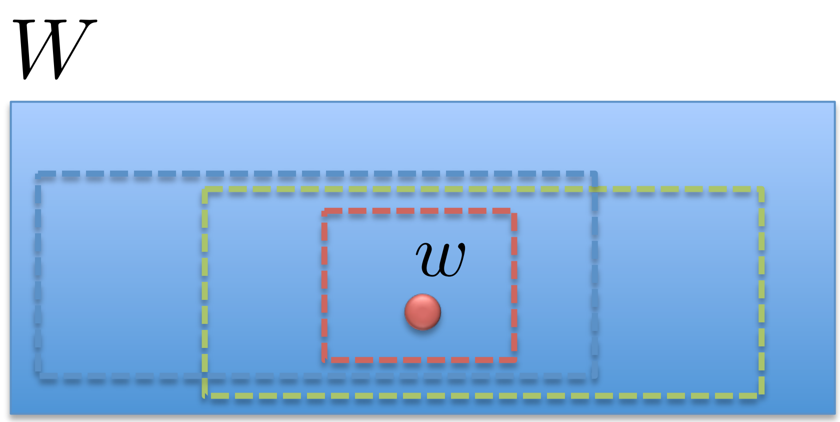
Three Axioms

1. Some information state true in w.



Three Axioms

1. Some information state true in w.
2. Each pair of information states true in w is entailed by a true information state true in w.

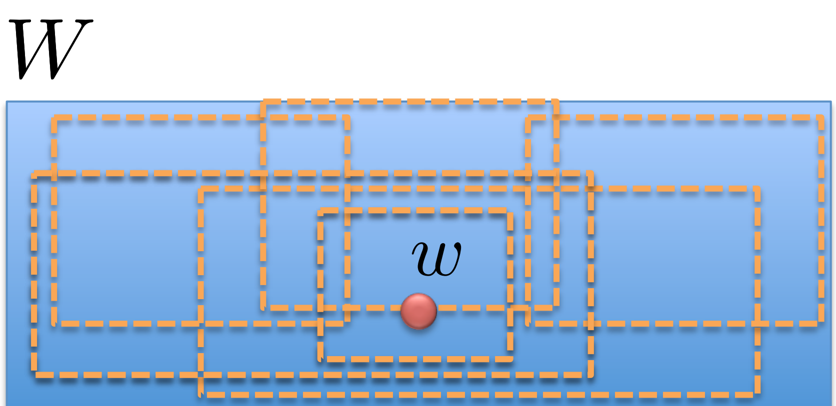


Three Axioms

1. Some information state true in w.
2. Each pair of information states true in w is entailed by a true information state true in w.
3. There are at most countably many information states.

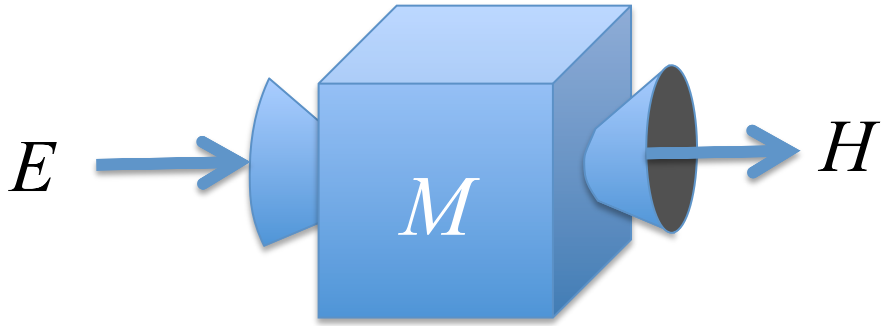
The Topology of Information

- $\mathcal{I}$ is a **topological basis** on W.
- Closing $\mathcal{I}$ under infinite disjunction yields a **topological space** on W.



Propositional Methods

- **Propositional methods** produce propositional conclusions in response to propositional information.



Deductive Success

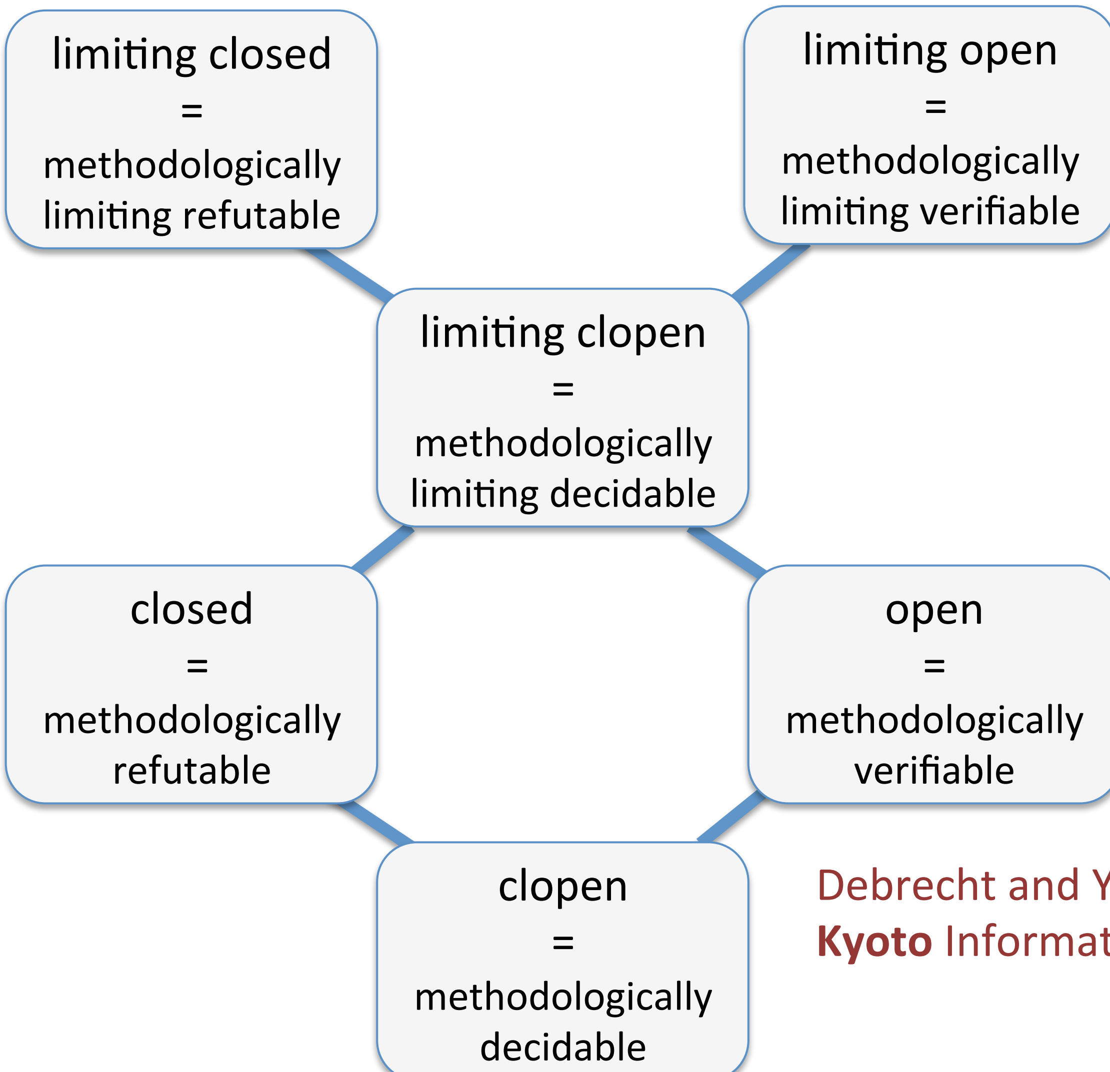
- A **verification method** for H is an method M such that in every world w:
  1. M converges infallibly to H if H is true in w.
  2. M always concludes W if H is false in w.

Inductive Success

- A **limiting verification method** for H is a method M such that in every world w:

  H is true in w iff M converges to some true H' that entails H.

Theorem.



Debrecht and Yamamoto,
Kyoto Informatics

TOPOLOGY AND STATISTICAL DEDUCTION

Can We Do the Same for Statistics?

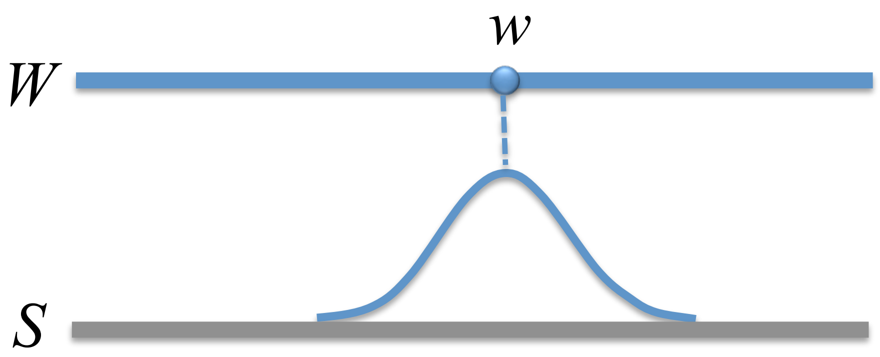
Kelly's topological approach...

"may be okay if the candidate theories are **deductively related** to observations, but when the relationship is **probabilistic**, I am **skeptical** ...".

Eliott Sober, *Ockham's Razors*, 2015

Statistical Worlds

- Probability measures over a sample space.



Statistical Verification

- A **statistical verification method** for H at **significance level** $\alpha > 0$:
  1. converges almost surely to H, if H is true.
  2. always concludes W with probability at least $1\text{-}\alpha$, if H is false.

- H is **statistically verifiable** iff H has a statistical verification method at each $\alpha > 0$.

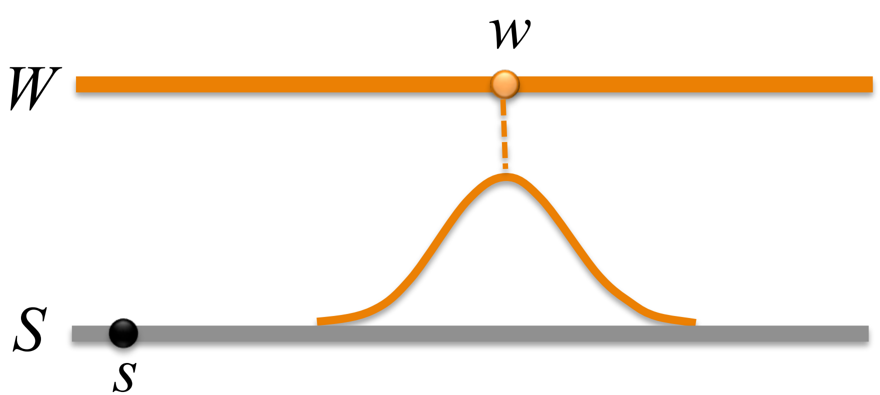
Statistical Verification in the Limit

- A **limiting statistical verification method** for H
  - converges almost surely to some H' entailing H iff H is true.

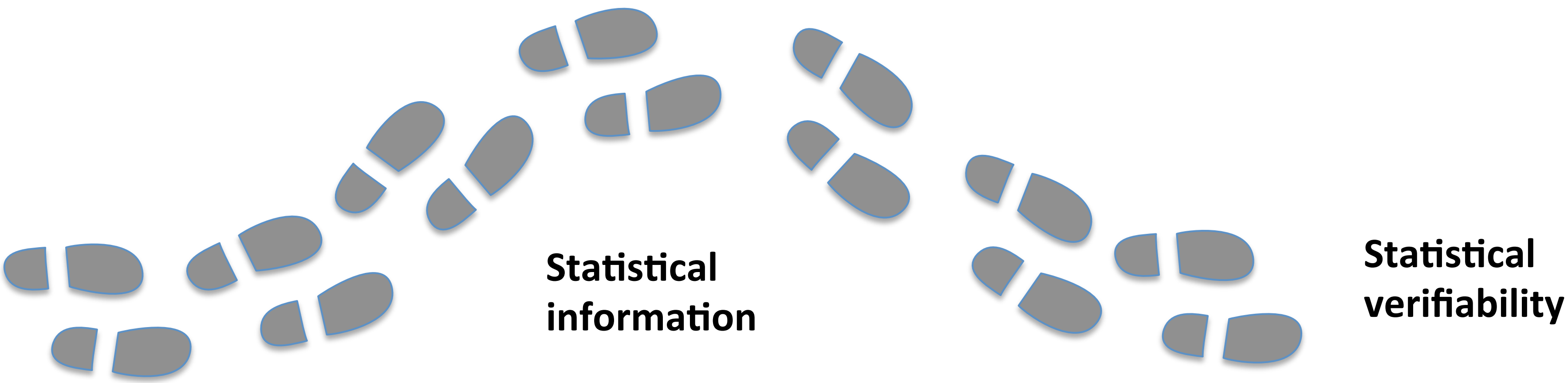
- H is **statistically verifiable in the limit** iff H has a limiting statistical verifier.

Recall the Worry

- It seems that the only statistical information state is W.



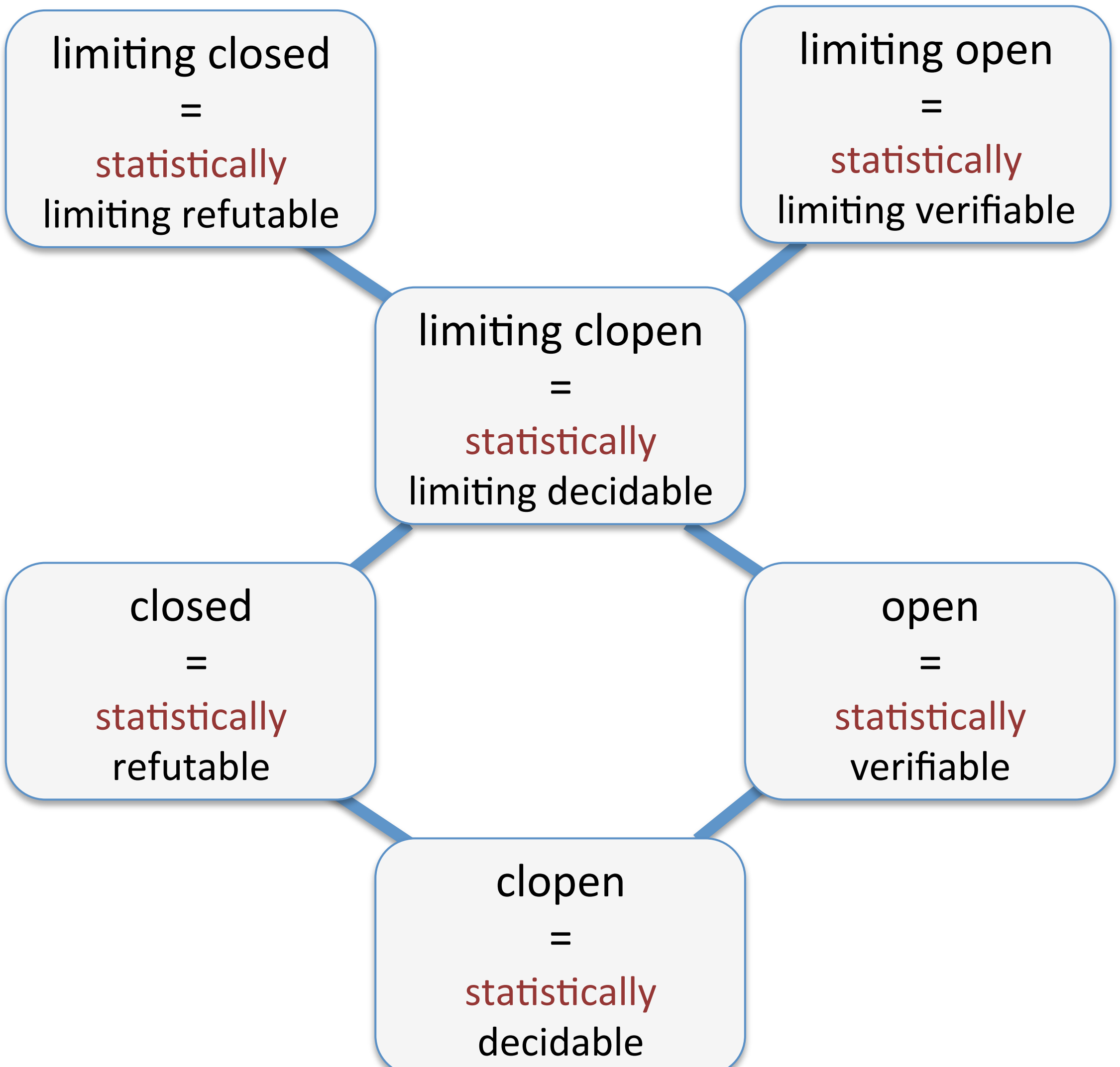
Side-step the Worry



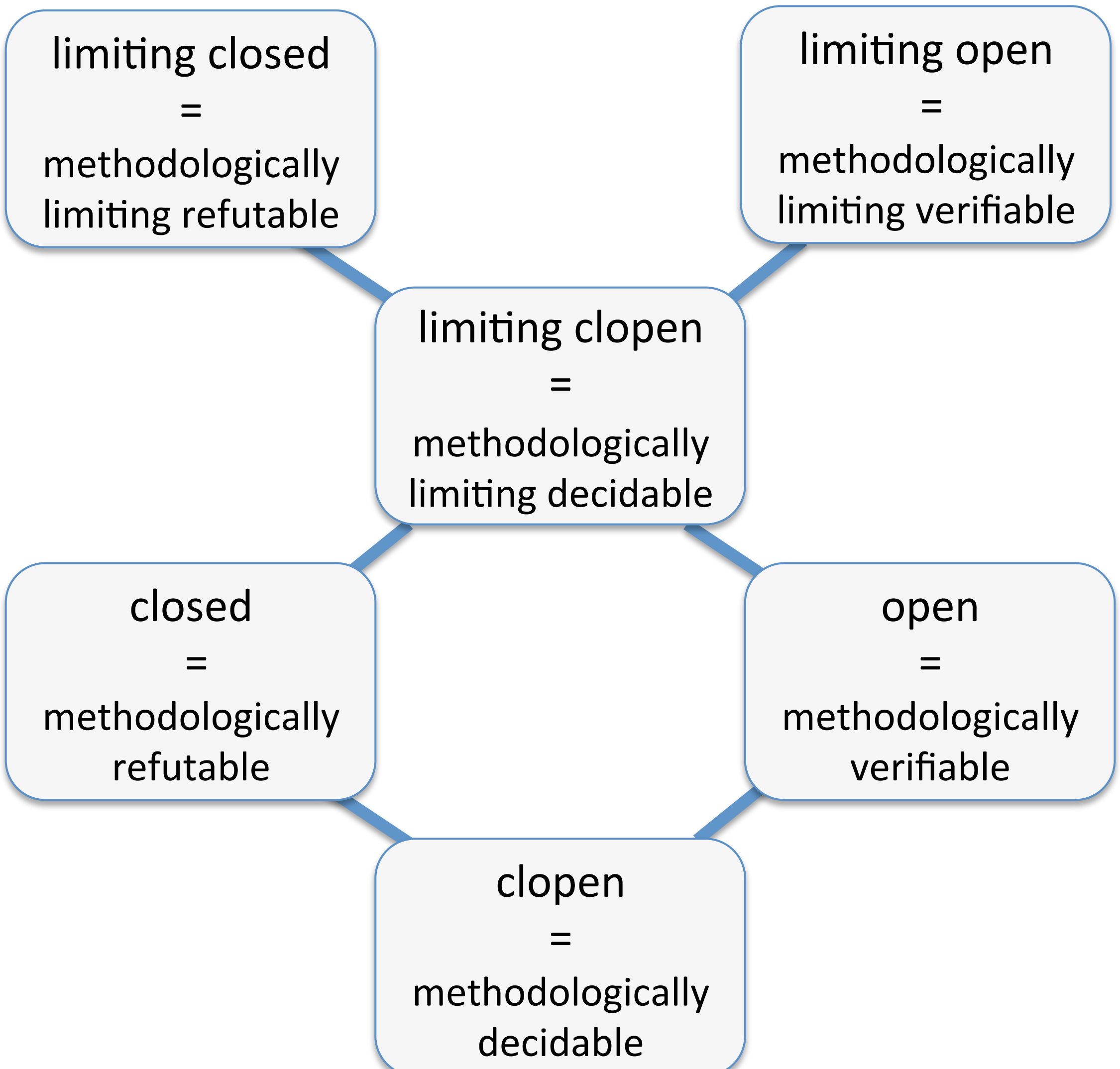
The Main Result

- Under some natural assumptions...
- there exists a unique and familiar topology on probability measures for which...

The Main Result



So in Both Logic and Statistics:



The Topological Bridge

The Topological Bridge

- Start with logical insights.
- Allow methods a small chance α of error.
- Obtain corresponding statistical insights

Thank you.
Come see the poster!