Theory Choice, Theory Change, and Inductive Truth-Conduciveness

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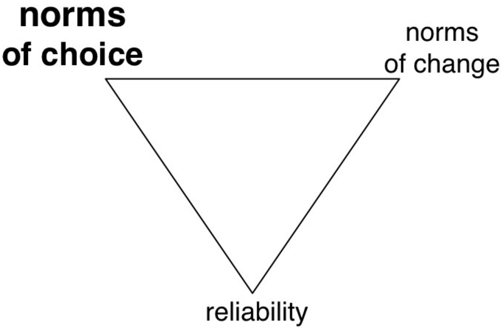
This talk is about ...



(1) the synchronic norms of theory ***choice***,

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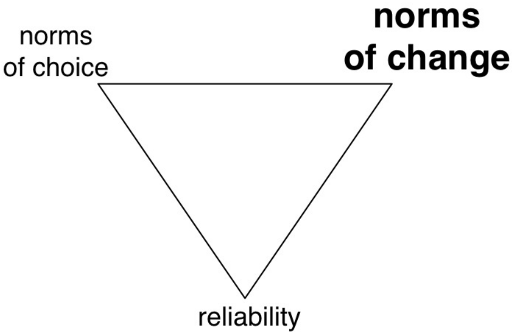

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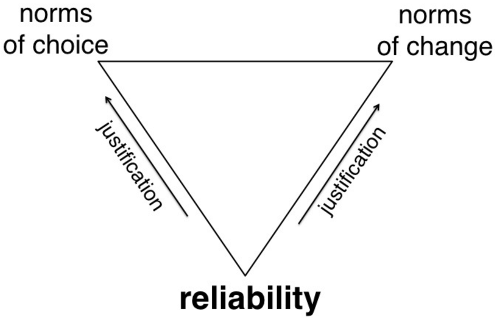
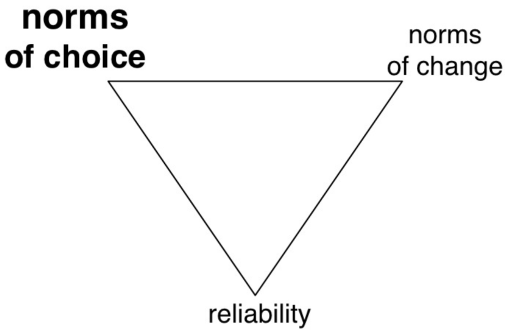


(1) the synchronic norms of theory ***choice***,

(2) the diachronic norms of theory ***change***, and

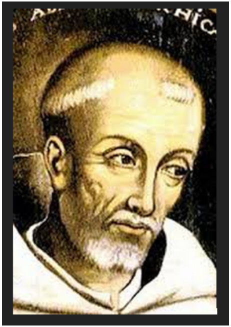
(3) the justification of (1-2) by ***reliability***, or ***truth-conduciveness***.

The Norms of Theory Choice



Synchronic norms of theory choice restrict the theories one can choose in light of given, empirical information.

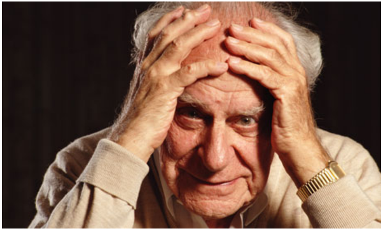
The Norms of Theory Choice: Simplicity



William of Ockham, 1287-1347

All things being equal, prefer ***simpler*** theories.

The Norms of Theory Choice: Falsifiability



Sir Karl Popper, 1902-1994

All things being equal, prefer more ***falsifiable*** theories.

The Norms of Theory Choice: Reliable?

Is the simpler / more falsifiable theory more *plausible*?

The Norms of Theory Choice: Reliable?

Is the simpler / more falsifiable theory more *plausible*?

Yes!

The Norms of Theory Choice: Reliable?

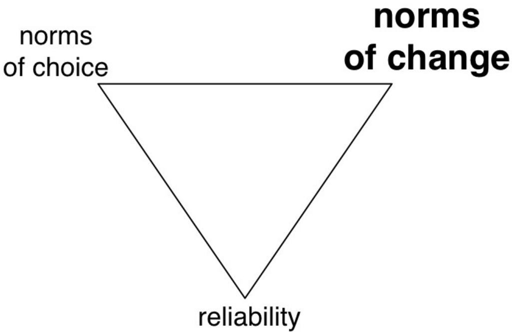
Does favoring the simple theory lead one to the truth *better* than alternative strategies?

The Norms of Theory Choice: Reliable?

Does favoring the simple theory lead one to the truth *better* than alternative strategies?

How could it, unless you *already know* that the world is simple?

The Norms of Theory Change



Diachronic norms of theory change restrict how one should change one's *current* beliefs in light of *new* information.

Norm of Minimal Change



Alchourrón, Gärdenfors, Makinson

To ***rationally*** accommodate new evidence, you ought to

1. add only those new beliefs and
2. remove only those old beliefs,

that are ***absolutely compelled*** by incorporation of new information.

Two Questions About the Norms of Theory Change

1. How are the norms of theory ***change*** related to the norms of theory ***choice***?

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   In his [book] The Web of Belief (1978), Quine has added more virtues that good theories should have: modesty, generality, refutability, and precision. Again, belief revision as studied so far has little to offer to reflect the quest for these intuitive desiderata. . . . It is a strange coincidence that the philosophy of science has focussed on the monadic (nonrelational) features of theory choice, while philosophical logic has emphasized the dyadic (relational) features of theory change. I believe that it is time for researchers in both fields to overcome this separation and work together on a more comprehensive picture (Rott, 2000, p. 15).

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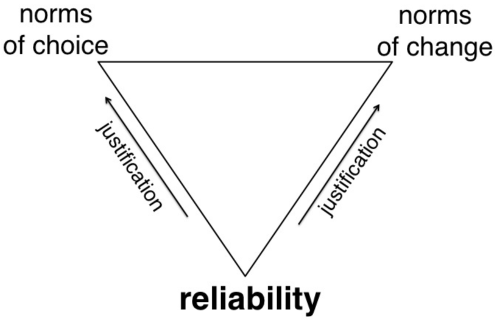
Two Questions About the Norms of Theory Change

1. How are the norms of theory ***change*** related to the norms of theory ***choice***?
2. Are the norms of theory change ***reliable***?

Epistemic Justification



Epistemic justification consists in showing that the norms are, in some sense, ***reliable***, or ***truth-conducive***.

Truth-conduciveness: Too Strong

Traditionally, truth-conduciveness has been ***too strictly*** conceived.

Truth-conduciveness: Too Strong

Traditionally, truth-conduciveness has been ***too strictly*** conceived.

> *...justifying an epistemic principle requires answering an epistemic question: why are parsimonious theories more likely to be true? (Baker, 2013)*

Truth-conduciveness: Too Strong

When your standards are too high you are led either to ***metaphysics***,

> *Nature is pleased with simplicity, and affects not the pomp of superfluous causes (Newton et al., 1833).*

Truth-conduciveness: Too Strong

. . . or ***despair***.

> *[N]o one has shown that any of these rules is more likely to pick out true theories than false ones. It follows that none of these rules is epistemic in character (Laudan, 2004).*

Truth-conduciveness: Too Strong

Theoretical virtues do not ***indicate*** the truth the way litmus paper indicates pH.

Truth-conduciveness: Too Strong

Inductive inferences made in accordance with the rationality principles are still subject to ***arbitrarily high chance of error***.

Truth-conduciveness: Too Strong

We can make progress if we cease to demand the impossible.

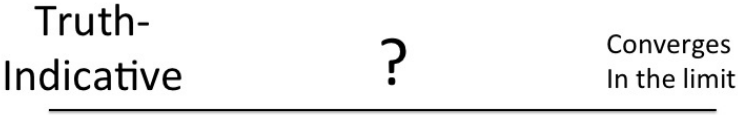
The fact that the truth of the predictions reached by induction cannot be guaranteed does not preclude a justification in a weaker sense (Carnap, 1945).

Truth-conduciveness: Too Weak

Truth-indicativeness is ***too strong*** a standard. But mere convergence to the truth in the limit is ***too weak*** to mandate any behavior in the short run.

> *Reichenbach is right ... that any procedure, which does not [converge in the limit] is inferior to his rule of induction. However, his rule ... is far from being the only one possessing that characteristic. The same holds for an infinite number of other rules of induction. ...* **Therefore we need a more general and stronger method for examining and comparing any two given rules of induction ...** *(Carnap, 1945)*

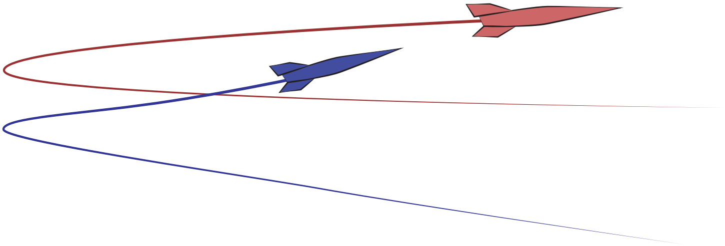
Truth-conduciveness: Just Right



Is there something in between?

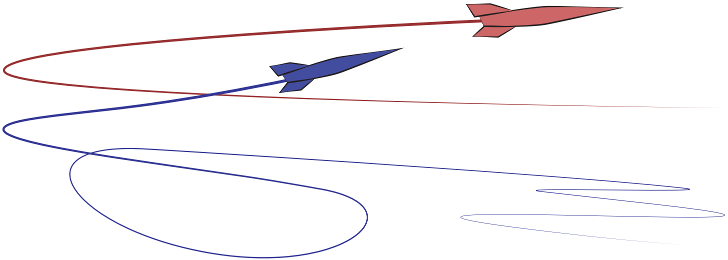
Refining Limiting Convergence

Pursuit of truth ought to be as direct as possible.

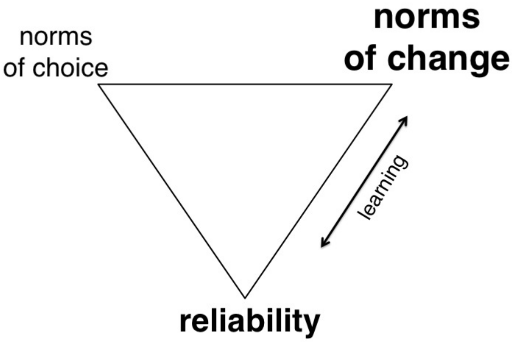


Refining Limiting Convergence

Needless cycles and reversals in opinion ought to be avoided.

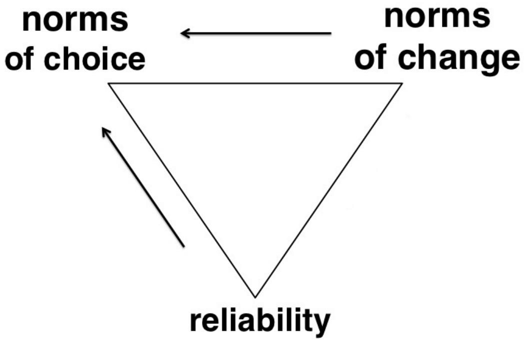


Summary



The ***truth-conduciveness norm*** of cycle-avoidance is equivalent to a weak ***norm of minimal change***, once limiting convergence is imposed.

Summary



Both principles necessitate a preference for ***simpler*** and ***more falsifiable*** theories.

Section 2

Topology as Epistemology

Topology as Epistemology

Related Approaches:

1. Vickers (1996)
2. Kelly (1996)
3. Luo and Schulte (2006)
4. Yamamoto and de Brecht (2010)
5. Baltag, Gierasimczuk, and Smets (2014)

Propositions and Possible Worlds

Let W be a set of possible worlds. A ***proposition*** is a set $P \subseteq W$. The contradictory proposition is $\varnothing$ and the necessary proposition is W.

- $P \wedge Q = P \cap Q$,
- $P \vee Q = P \cup Q$,
- $\neg P = W \backslash P$, and
- P entails Q iff $P \subseteq Q$.

Information States

Let $\mathcal{I} \subseteq \mathfrak{P}(W)$ be the set of possible ***information states***. Then the set of all information states compatible with world w is:

$$\mathcal{I}(w) = \{E \in \mathcal{I} : w \in E\}.$$

Definition 1

$\mathcal{I}$ is an ***information basis*** iff the following are satisfied:

I1. $\bigcup \mathcal{I} = W$;

I2. If $A, B \in \mathcal{I}(w)$ then $A \cap B \in \mathcal{I}(w)$;

I3. $|\mathcal{I}| \leqslant \omega$.

Verifiable Propositions

Definition 2 (Verifiable Propositions)

A proposition P is ***verifiable*** iff for every world $w \in P$ there is some information state $E \in \mathcal{I}(w)$ such that E entails P.

It follows from this definition that the verifiable propositions $(W, \mathcal{V})$ form a ***topology***.

Verifiable Propositions

The verifiable propositions are closed under ***finite*** conjunction. You can verify finitely many sunrises,

   ...

But not that it will rise every morning.

[Conditionally] Falsifiable Propositions

Definition 3

A proposition P is ***falsifiable*** iff $\neg P$ is verifiable.

Definition 4

P is ***conditionally falsifiable***

1. iff P is the intersection of an open and closed set (locally closed);
2. A implies that A will be refutable.

A Translation Key

To translate between topology and epistemology:

1. basic open set $\equiv$ information state.
2. open set $\equiv$ verifiable proposition.
3. closed set $\equiv$ falsifiable proposition.
4. clopen set $\equiv$ decidable proposition.
5. locally closed set $\equiv$ conditionally refutable proposition.

The Topology of the Problem of Induction

The bread, which I formerly ate, nourished me ... but does it follow, that other bread must also nourish me at another time, and that like sensible qualities must always be attended with like secret powers? The consequence seems nowise necessary (Enquiry Concerning Human Understanding).

Sierpinski Space

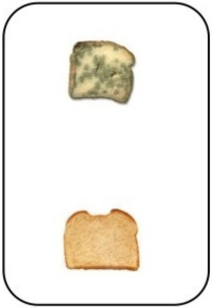
Suppose we have two worlds.

Sierpinski Space

Suppose we have two worlds.

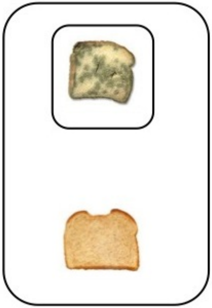
Sierpinski Space

If bread always nourishes, we can never rule out that one day it will stop nourishing.



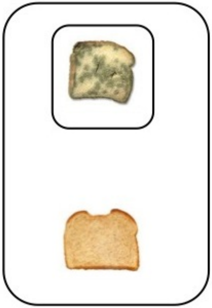
Sierpinski Space

If someday bread will cease to nourish, this will be verified.



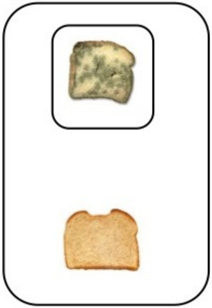
Sierpinski Space

This structure defines the *Sierpinski space*, a simple topological space.



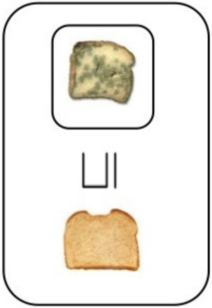
Sierpinski Space

Note that the bottom world *entails* that its complement will never be refuted.



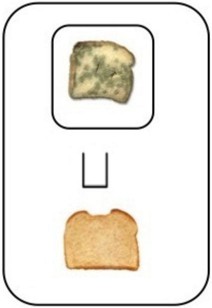
The Specialization Order

Let $w \sqsubseteq v$ iff $\mathcal{O}(w) \subseteq \mathcal{O}(v)$ i.e. all information consistent with w is consistent with v.



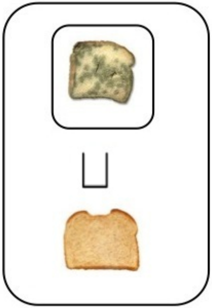
The Specialization Order

Let $w \sqsubset v$ if $w \sqsubseteq v$ but $v \not\sqsubseteq w$.



The Specialization Order

That defines the ***specialization order*** over points in the space.



Section 3

Empirical Simplicity

Topological Closure

Definition 5

The ***closure*** of a proposition A is the set of all worlds where A is never refuted:

$$\overline{A} = \{w : \text{Every } E \in \mathcal{I}(w) \text{ is consistent with } A\}.$$

So for every world w, $\overline{\{w\}} = \{v : v \preceq w\}$.

The Problem of Induction: Defined

Definition 6

Say that A ***poses the problem of induction***

1. iff $A \subseteq \overline{\neg A}$;
2. A entails that $\neg A$ will never be refuted.

Definition 7

Say that A ***poses the problem of induction w.r.t*** B

1. iff $A \subseteq \overline{B} \backslash B$;
2. A entails that B is false, but will never be refuted.

Definition 8 (Simplicity)

Say that A is ***simpler*** than B, written $A \prec B$, iff A poses the problem of induction w.r.t B.

Popper and Simplicity

The epistemological questions which arise in connection with the concept of simplicity can all be answered if we equate this concept with degree of falsifiability (Popper, 1959).

Popper and Simplicity

A proposition P is ***more falsifiable*** than Q if and only if all information that falsifies Q falsifies P.

Equivalently, all information consistent with P is consistent with Q.

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Equivalently, all information consistent with P is consistent with Q.

So if P is true, Q will never be refuted. Therefore $P \subseteq \overline{Q}$.

Popper and Simplicity: Glymour's Tack-On Objection

An awkward consequence of Popper's view: logically stronger propositions are simpler.

But intuitively, the conjunction of GR with some irrelevant hypothesis H is not simpler than GR.

Definition 9 (The Simplicity Relation)

P is ***simpler than*** Q, written $P \prec Q$,

1. iff $P \subseteq \overline{Q} \setminus Q$,
2. iff P entails that Q is false, but will never be refuted,
3. iff P has a problem of induction with Q,
4. iff P is more falsifiable than, but incompatible with, Q.

P is ***at least as simple as*** Q, written $P \preceq Q$ iff $P \prec Q$ or $P = Q$.

Simplest Propositions

Definition 10

Define the following notation for the set of worlds simpler than P

$$P_{\prec} = \bigcup \{Q \prec P : Q \subseteq W\}$$

Say that P is **minimal** in simplicity iff $P_{\prec} = \varnothing$.

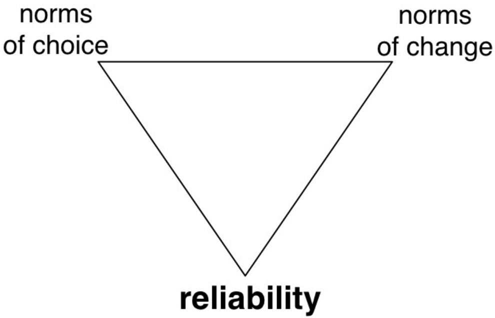
Proposition 1

P is minimal in the simplicity order iff P is closed (falsifiable).

Section 4

Reliability

Reliability



In this section we develop several learning-theoretic notions of ***reliability,*** or ***truth-conducivenes***. We start with ***limiting convergence***, and then develop some refinements.

Empirical Problems

Definition 11

An ***empirical problem context*** is a triple $\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$.

- W is the set of possible worlds.
- $\mathcal{I}$ is an information basis.
- $\mathcal{Q}$ is a *question* that partitions W into countably many *answers*.

Let $\mathcal{Q}(w)$ be the answer true at w and $\mathcal{Q}(P) = \bigcup_{w \in P} \mathcal{Q}(w)$.

Limiting Convergence

Definition 12

An ***empirical method*** is a function $\lambda : \mathcal{I} \to \mathcal{Q}^{\omega}$.

Definition 13

A method λ ***solves*** $\mathfrak{P}$ ***in the limit*** iff for all $w \in W$, there is ***locking information*** $E \in \mathcal{I}(w)$, such that for all $F \in \mathcal{I}(w)$, $\lambda(E \cap F) = \mathcal{Q}(w)$.

Definition 14

A problem $\mathfrak{P}$ is ***solvable in the limit*** iff there exists λ that solves it in the limit.

Solvable Problems Characterized

Proposition 2 (de Brecht and Yamamoto (2009))

$\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$ *is solvable in the limit method iff each* $Q \in \mathcal{Q}$ *is a countable union of locally closed sets.*

Consistent Methods

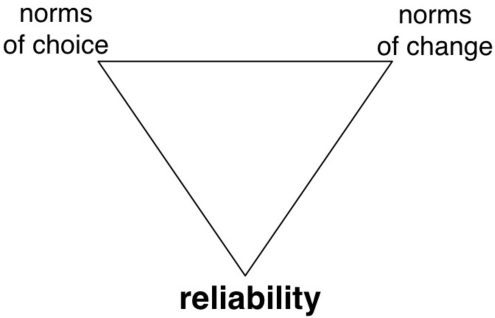
Definition 15 (Consistency)

Method λ is ***consistent*** iff for all $E \in \mathcal{I}$, $\lambda(E) \subseteq \mathcal{Q}(E)$.

Proposition 3

Every solvable problem is solvable by a consistent method.

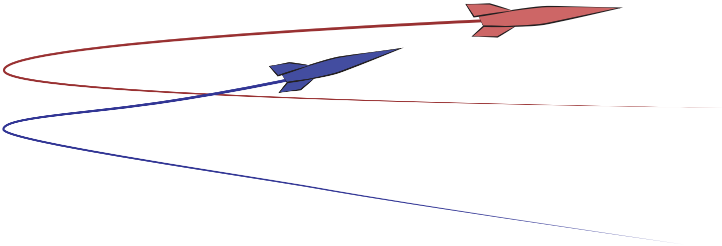
Refining Limiting Convergence



Now we develop some norms of **optimally direct** convergence, that refine limiting convergence.

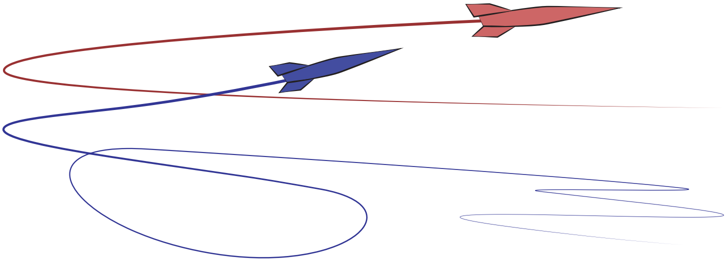
Refining Limiting Convergence

Pursuit of truth ought to be as direct as possible.



Refining Limiting Convergence

Needless cycles and reversals in opinion ought to be avoided.



Cycles and Reversals

Definition 16 (Retractions)

A **retraction sequence** is a sequence of elements of $\mathcal{Q}^\omega$, $(A_i)_{i=1}^n$ such that $A_{i+1} \nsubseteq A_i$ for $1 \leqslant i < n$.

Definition 17 (Reversals)

A **reversal sequence** is a retraction sequence $(A_i)_{i=1}^n$ such that $A_{i+1} \subseteq A_i^c$ for $1 \leqslant i < n$.

Definition 18 (Cycles)

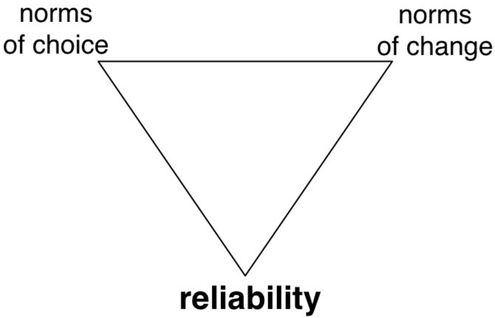
A **cycle sequence** is a reversal sequence $(A_i)_{i=1}^n$ such that $A_n \subseteq A_1$.

Comparing Retraction Sequences

Definition 19 (Comparing Conjecture Sequences)

Suppose $\sigma = (A_i)_{i=1}^{n}$ and $\delta = (B_i)_{i=1}^{n}$ are retraction sequences. Set $\sigma \leqslant \delta$ iff $A_i \subseteq B_i$ for $1 \leqslant i \leqslant n$.

Refining Limiting Convergence: Avoiding Cycles



We focus now on ***avoiding cycles*** as a norm of truth-conducive performance.

Avoiding Cycles

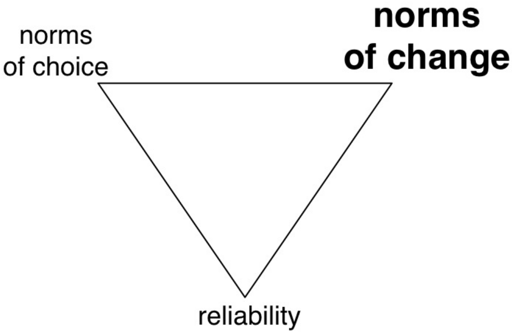
Definition 20 (Cycle-Free Learning)

Method λ is ***cycle-free*** iff there exists no nested sequence of non-empty information states:

$$e = (E_i)_{i=1}^{n},$$

such that $\lambda(e) = (\lambda(E_i))_{i=1}^{n}$ is a cycle sequence.

Norms of Theory Change



We now state some principles of ***rational theory change***, from belief revision.

Norms of Theory Change

Definition 21 ("No induction, without refutation.")

A method λ satisfies ***conditionalization*** iff for all $E, F \in \mathcal{I}$,

$$\lambda(E) \cap \mathcal{Q}(E \cap F) \subseteq \lambda(E \cap F).$$

Definition 22 ("No retraction, without refutation.")

A method λ is ***rationally monotone*** iff

$$\lambda(E \cap F) \subseteq \lambda(E) \cap \mathcal{Q}(E \cap F)$$

for all $E, F \in \mathcal{I}$ such that $\lambda(E) \cap \mathcal{Q}(E \cap F) \neq \varnothing$.

The previous two principles are both weakened by the following:

Definition 23 ("No reversal, without refutation.")

A method λ is ***reversal monotone*** iff

$$\lambda(E \cap F) \cap \lambda(E) \neq \varnothing \text{ whenever } \lambda(E) \cap \mathcal{Q}(E \cap F) \neq \varnothing.$$

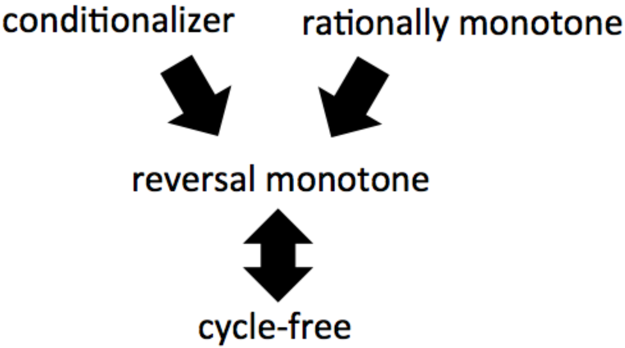
Norms of Theory Change and Truth-Conduciveness.

Proposition 4

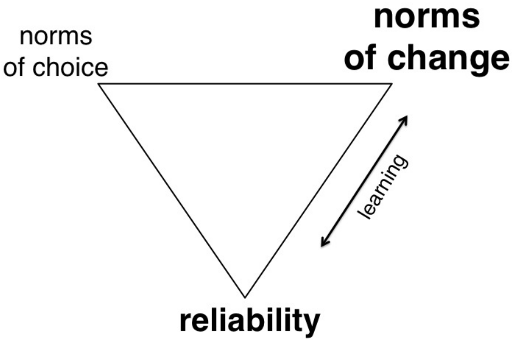
If λ is a consistent solution to $\mathfrak{P}$, then λ is cycle-free iff λ is reversal monotone.

So once the requirement of learning is imposed, cycle-free learning (a truth-conduciveness concept) is ***equivalent*** to a weak principle of theory change.

Norms of Theory Change and Truth-Conduciveness.

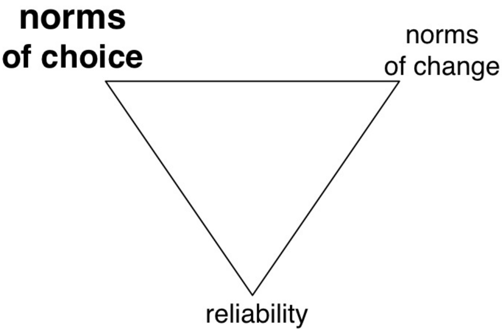


A Truth-Conducive Norm of Theory Change



Proposition 4 delivers on our promise to provide a ***truth-conducive justification*** for the ***norms of theory change***

The Norms of Theory Choice



We now turn to two traditional theory ***choice*** norms: ***simplicity*** and ***falsifiability***.

Definition 24 (Ockham Methods)

A method λ is ***Ockham*** iff $\lambda(E)$ is minimal in $\preceq$ for all $E \in \mathcal{I}$.

The Norms of Theory Choice: Falsifiability

Definition 25 (Popperian Methods)

λ is ***Popperian*** iff $\lambda(E)$ is closed (falsifiable) in E for all $E \in \mathcal{I}$.

As an immediate consequence of Proposition 1:

Proposition 5

λ is Popperian iff λ is Ockham.

Proposition 6

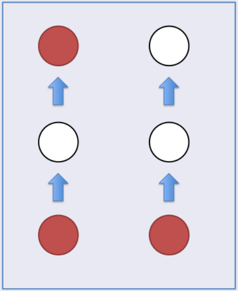
If λ is a cycle-free solution, then λ is Ockham.

Ockham and Cycle Avoidance

Proposition 7

If λ is a cycle-free solution, then λ is Ockham.

Sketch.



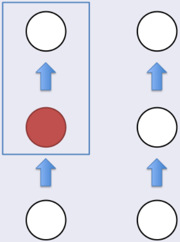
Suppose you violate Ockham's razor. □

Ockham and Cycle Avoidance

Proposition 8

If λ is a cycle-free solution, then λ is Ockham.

Sketch.



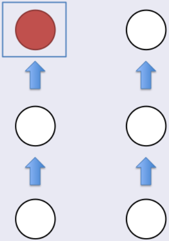
You reverse on further information, though your first conjecture is not refuted. □

Ockham and Cycle Avoidance

Proposition 9

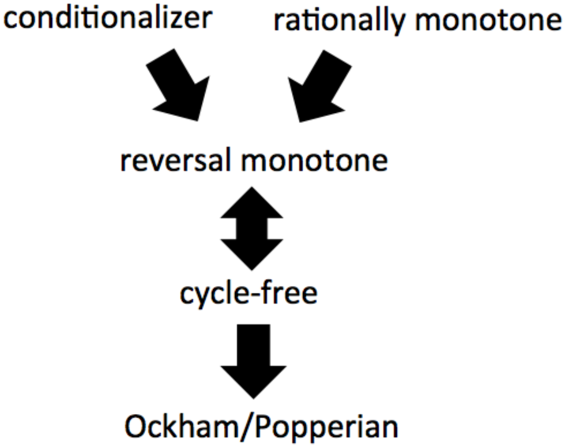
If λ is a cycle-free solution, then λ is Ockham.

Sketch.

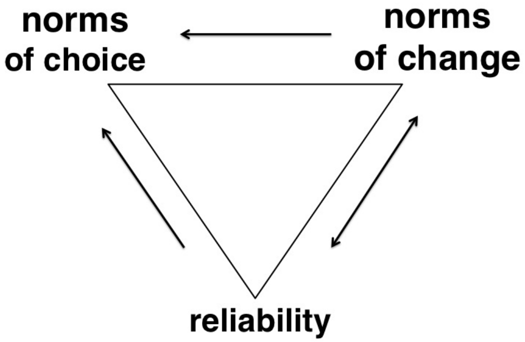


On even further information, you are forced into a cycle. □

The Norms of Theory Choice, Justified



The Norms of Theory Choice, Justified



The previous proposition gives a ***truth-conducive justification*** for the norms of theory ***choice*** and connects them to the norms of theory ***change***.

Avoiding Cycles: Feasibility

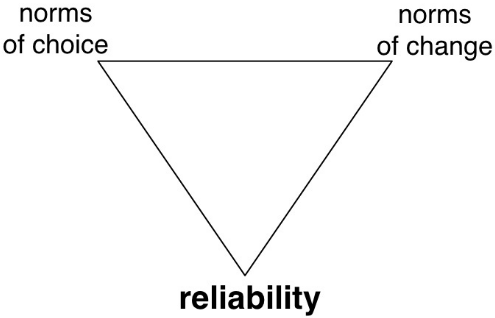
Proposition 10

If $\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$ is a solvable problem then there is $\mathfrak{P}' = (W, \mathcal{I}, \mathcal{Q}')$ such that $\mathcal{Q}'$ refines $\mathcal{Q}$ and $\mathfrak{P}'$ is solved in the limit by a cycle free method.

Corollary 26

If $\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$ is a solvable problem, then there is $\mathfrak{P}' = (W, \mathcal{I}, \mathcal{Q}')$ such that $\mathcal{Q}'$ refines $\mathcal{Q}$ and $\mathfrak{P}'$ is solved in the limit by an Ockham method.

Refining Limiting Convergence: Minimizing Reversals



We focus now on ***minimizing reversals*** as a norm of truth-conducive performance.

Minimizing Reversals

Definition 27 (Forcible Sequences)

A reversal sequence $\delta = (A_i)_{i=1}^n$ is ***forcible*** iff for every λ that solves $\mathfrak{P}$, there is a nested sequence of information states $e = (E_i)_{i=1}^n$ such that $\lambda(e)$ is a reversal sequence and $\lambda(e) \leqslant \delta$.

Definition 28

We say that a method is ***reversal-optimal*** if all of its reversal sequences are forcible.

Forcible Sequences

Theorem 29

If $\mathfrak{P}$ is solvable, then reversal sequence $a = (A_0, \dots, A_n)$ is forcible *iff*

$$A_0 \cap \overline{A_1 \cap \overline{\dots \cap \overline{A_{n-1} \cap \overline{A_n}}}} \neq \varnothing.$$

Definition 30

$\mathfrak{P}$ is ***sensible*** iff each $A \in \mathcal{Q}$ is locally closed, and for all $A, B \in \mathcal{Q}^\omega$, $A \cap \overline{B} \neq \varnothing$ entails $A \subseteq \overline{B}$.

Corollary 31

If $\mathfrak{P}$ is sensible, then reversal sequence $a = (A_0, \dots, A_n)$ is forcible *iff*

$$A_0 \prec A_1 \prec \dots \prec A_n.$$

Minimizing Reversals

Definition 32 (Patience)

A method λ is ***patient*** iff for all $E \in \mathcal{I}$ and $Q \subseteq \mathcal{Q}(E)$, there is $Q' \subseteq \lambda(E)$ such that $Q' \cap \overline{Q} \neq \varnothing$.

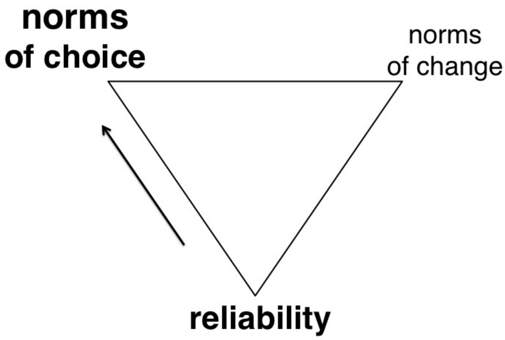
Theorem 33

*If $\mathfrak{P}$ is sensible, then λ is **patient** iff $\lambda(E)$ is co-initial in $\leq$.*

Theorem 34

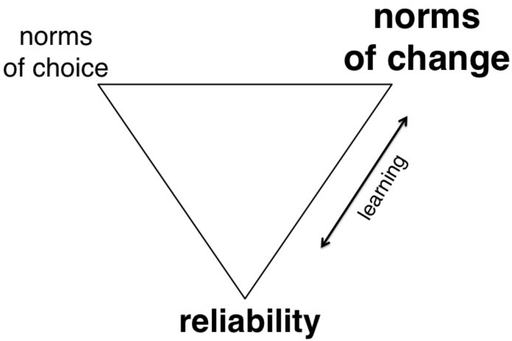
Every reversal optimal solution is patient.

Minimizing Reversals and a Norm of Theory Choice



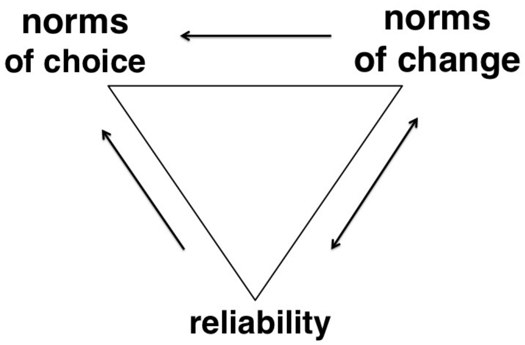
The previous proposition shows that the ***truth-conduciveness norm*** of minimizing reversals entails an Ockham ***norm of theory choice***

Summary



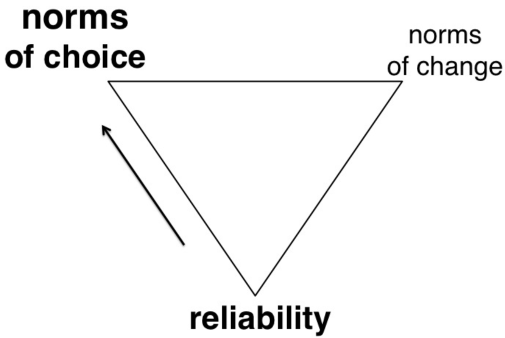
The ***truth-conduciveness norm*** of cycle-avoidance is equivalent to a weak ***norm of rational theory change***, once limiting convergence is imposed.

Summary



Both principles entail the ***theory choice norm*** that all conjectures be minimal in the simplicity order (falsifiable).

Summary



Furthermore, the ***truth-conduciveness norm*** of reversal minimization entails the horizontal-Ockham ***norm of theory choice.***

Summary

1. ***Reliability*** and the norms of ***choice***.
   - Avoiding cycles entails Ockham's razor (falsificationism).
   - Minimizing reversals entails patience.
2. ***Reliability*** and the norms of ***change***.
   - Avoiding cycles is equivalent to a weak principle of theory change, one the requirement of convergence is imposed.
3. Norms of ***choice*** and norms of ***change.***
   - The principles of rational change all entail Ockham's razor (falsificationism).

Summary

Thank you!

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