

# On Falsifiable Statistical Hypotheses

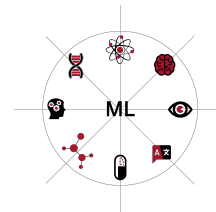
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University of Tübingen

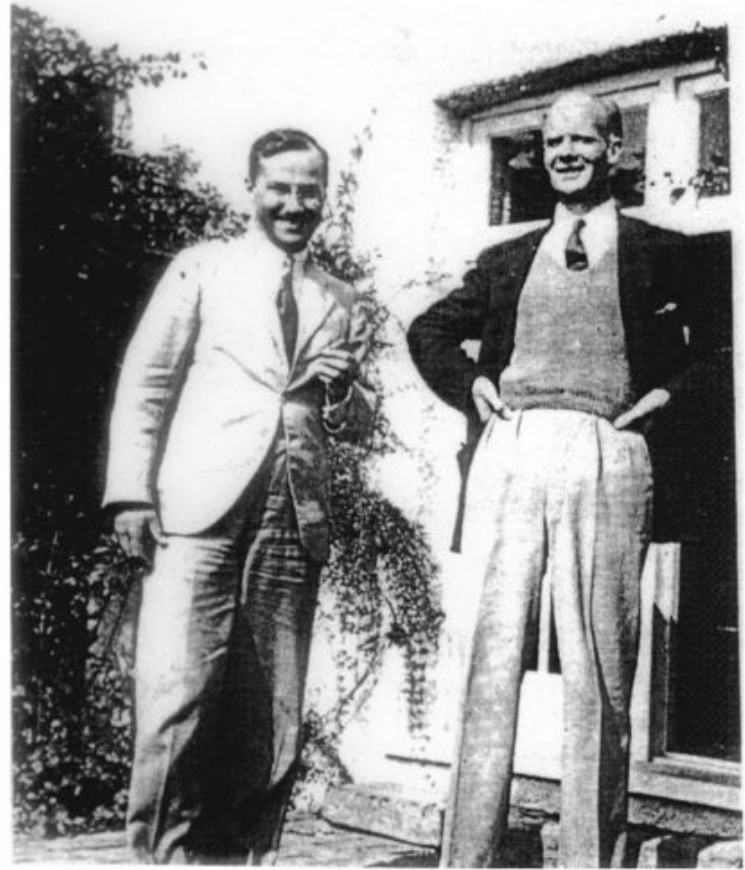
2022 Formal Epistemology Workshop  
Irvine, CA

EBERHARD KARLS  
UNIVERSITÄT  
TÜBINGEN



# Some Trivial Observations

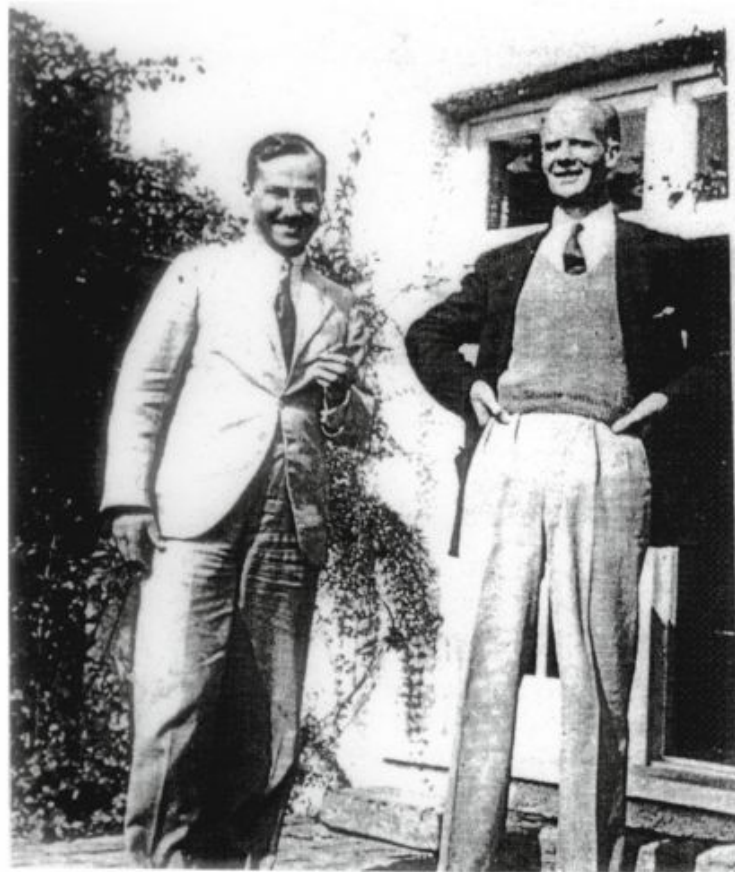
At first glance, frequentist practice looks falsificationist.



*Jerzy Neyman and Egon Pearson*

# Some Trivial Observations

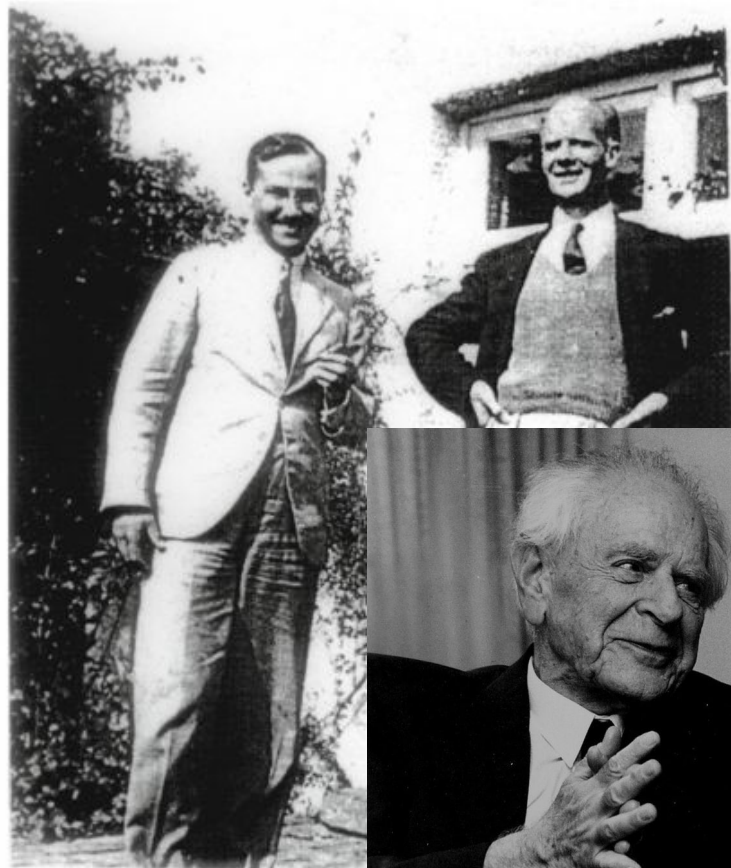
Consider a test of a sharp null hypothesis (e.g. ‘the coin is fair’) that “rejects” upon observing an event that would be highly improbable if the null were true and otherwise “accepts”.



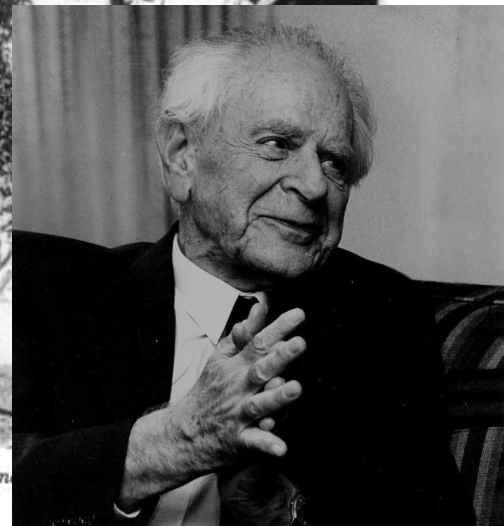
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# Some Trivial Observations

- If null is true, low chance of rejecting in error :: rejecting a universal hypothesis when confronted with a countervailing instance.
- If null is (subtly) false, high chance of accepting in error :: accepting a universal hypothesis when confronted with many confirming instances.



Jerzy Neyman and



# Statistician's Self-Conception

“ . . . the hypothesized model makes certain probabilistic assumptions, from which other probabilistic implications follow deductively. Simulation works out what those implications are, and tests check whether the data conform to them. Extreme p-values indicate that the data violate regularities implied by the model, or approach doing so.

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Gelman, Andrew, and Cosma Rohilla Shalizi. "Philosophy and the practice of Bayesian statistics." *British Journal of Mathematical and Statistical Psychology* 66.1 (2013): 8-38.

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“ . . . If these were strict violations of deterministic implications, we could just apply modus tollens to conclude that the model was wrong; as it is, we nonetheless have evidence and probabilities. Our view of model checking, then, is firmly in the long hypothetico-deductive tradition, running from Popper (1934/1959) back through Bernard (1865/1927) and beyond (Laudan, 1981).”



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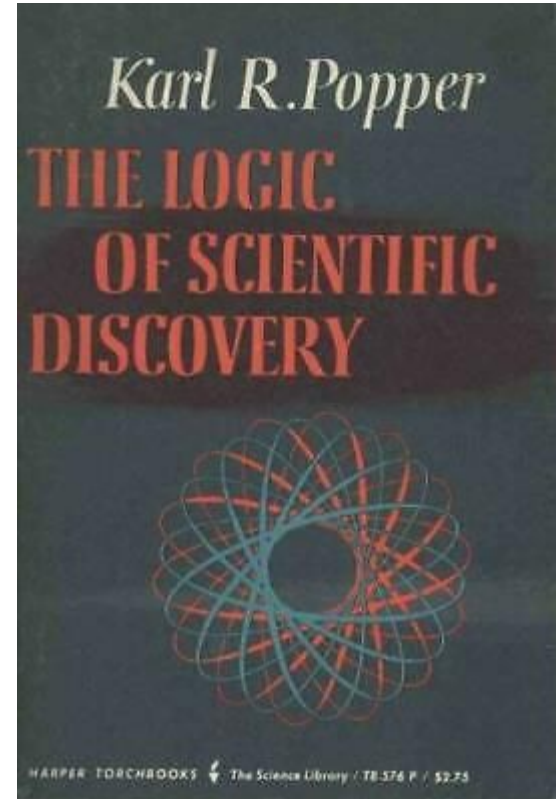
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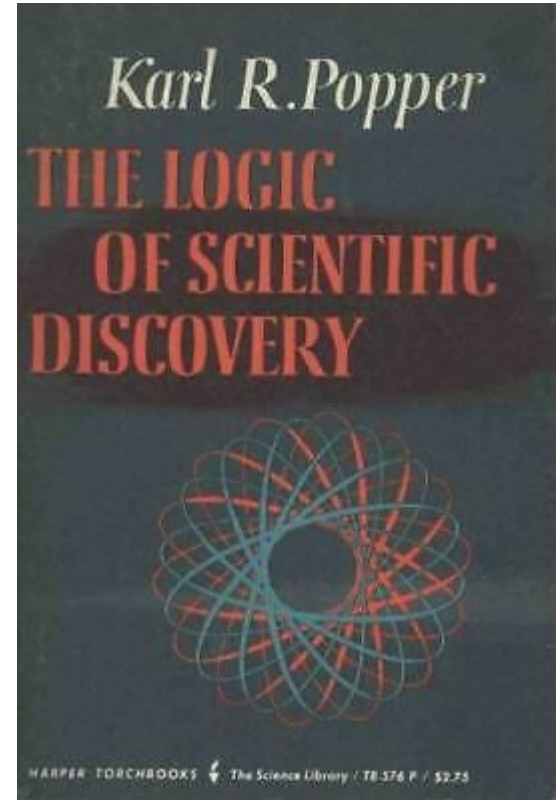
# Popper's Stumbling Block

“... although probability statements play such a vitally important role in empirical science, they turn out to be in principle impervious to strict falsification. Yet this very stumbling block will become a touchstone upon which to test my theory, in order to find out what it is worth” (Popper, 1959, p. 133).



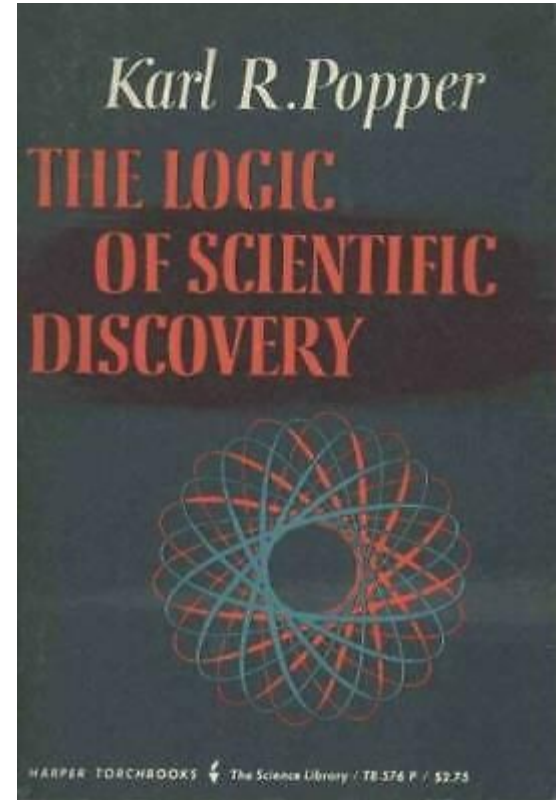
# Popper's Stumbling Block

“... a physicist is usually quite well able to decide whether he may for the time accept some particular probability hypothesis as ‘empirically confirmed’, or whether he ought to reject it as ‘**practically falsified**’, i.e. as useless for the purposes of prediction. ... ” (Popper, 1959, p. 182).



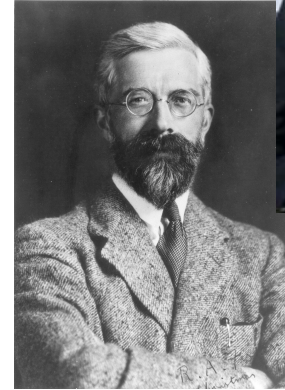
# Popper's Stumbling Block

“It is fairly clear that this ‘practical falsification’ can be obtained only through a **methodological decision** to regard highly improbable events as ruled out ... But by what right can they be so regarded? Where are we to draw the line? Where does this “high improbability” begin?” (Popper, 1959, p. 182).



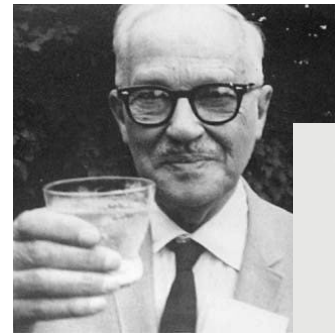
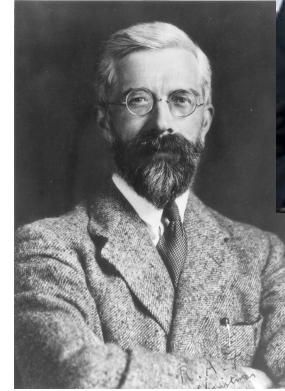
# Proposals and Critiques

- Fisher (1959) and Gillies (1971): just pick some canonical event that is highly improbable if the hypothesis is true.



# Proposals and Critiques

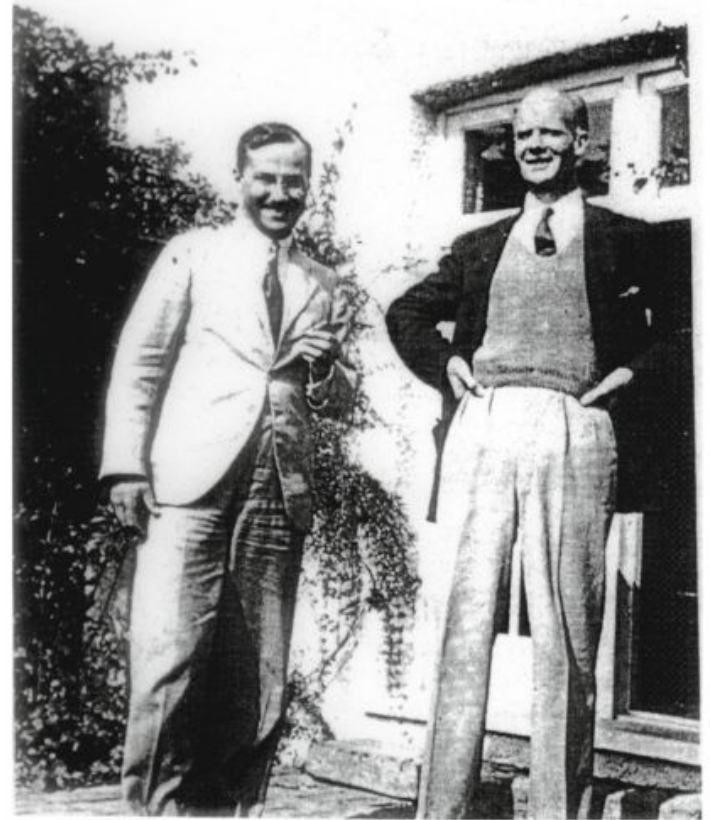
- Fisher (1959) and Gillies (1971): just pick some canonical event that is highly improbable if the hypothesis is true.
- Neyman (1952) and Redhead (1974): there is no unique convention satisfying this property; indeed competing conventions may give conflicting verdicts on *every* sample.





# Proposals and Critiques

- Neyman and Pearson (1933): not enough to pick an event that is improbable if the hypothesis is true; should also be maximally probable if the hypothesis is false.
- Typically no way to choose an event with “uniformly” high probability in all the possibilities in which the hypothesis is false: one must always favor some alternative over others.



*Jerzy Neyman and Egon Pearson*

# Proposals and Critiques

Is there no non-arbitrary answer to this question?



# Changing Tack: from Refutation to Refutability

- The question ‘which hypotheses are **refutable**?’ is at least as important as ‘when should data be taken to have **refuted** a hypothesis?’
- There does not have to be a univocal answer to the latter question; if there were it would no longer have a methodological/conventional aspect.



# Changing Tack: from Refutation to Refutability

“Popper demands in science refutability, not refutation” (Redhead, 1974).



# Changing Tack: from Synchronic to Diachronic

- Excessive focus on the synchronic virtues of falsification methods tends to obscure the social dimensions of our methodological decisions.
- A good methodological convention should, if adopted, support a pattern of successful replication by independent investigators.



# Why Not Severe Testability?

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- $H$  is severely tested by  $E$  if, were  $H$  false, evidence less favorable to  $H$  would probably have been observed.
- Why not falsifiable = severely testable?
- Too radical a revision: sharp null hypotheses like 'the coin is fair' would not be falsifiable.



# The Guiding Analogy

“Classical” Phil. Science

Statistics

<i>Error Avoidance.</i> Output conclusions are true.	

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<i>Monotonicity.</i> Logically stronger inputs yield logically stronger conclusions.	<i>Monotonicity in Chance.</i> If $H$ is [ true   false], then, for any sample sizes $n_1 < n_2$ , the chance of correctly [accepting   rejecting] $H$ <b>does not decrease</b> .
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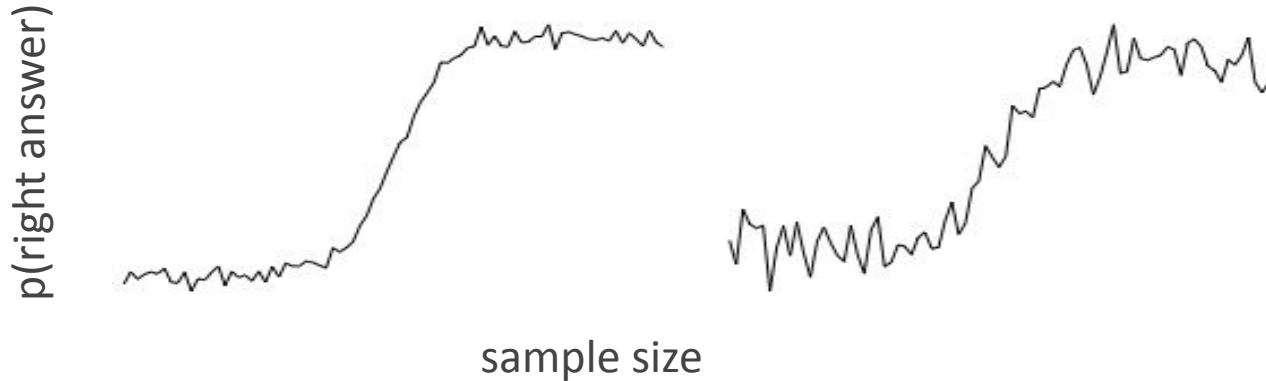
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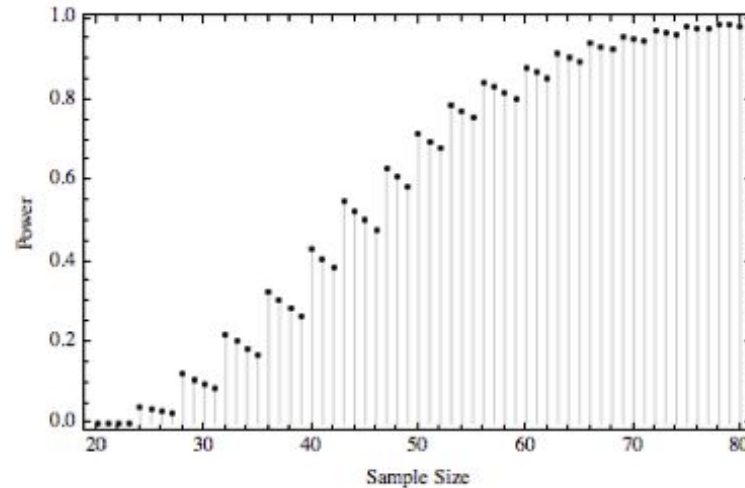
# $\alpha$ -Monotonicity

If  $H$  is [ true | false], then, for any sample sizes  $n_1 < n_2$ , the chance of correctly [accepting | rejecting]  $H$  decreases by no more than  $\alpha$ .



# $\alpha$ -Monotonicity

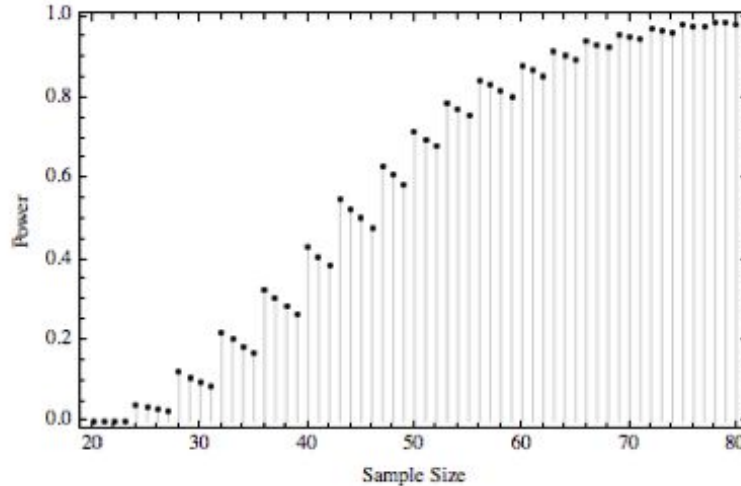
Surprisingly, many textbook statistical tests are not even  $\alpha$ -progressive.



Chernick, Michael R., and Christine Y. Liu. (2002) "The Saw-Toothed Behavior of Power Versus Sample Size and Software Solutions." *The American Statistician* 56, no. 2: 149–55. <https://doi.org/10.1198/000313002317572835>.

# $\alpha$ -Monotonicity

Collecting a larger sample may be a **bad** idea!



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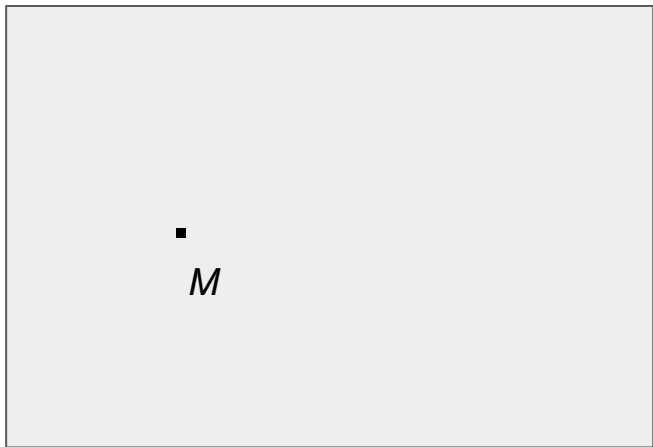
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# The Problem Context

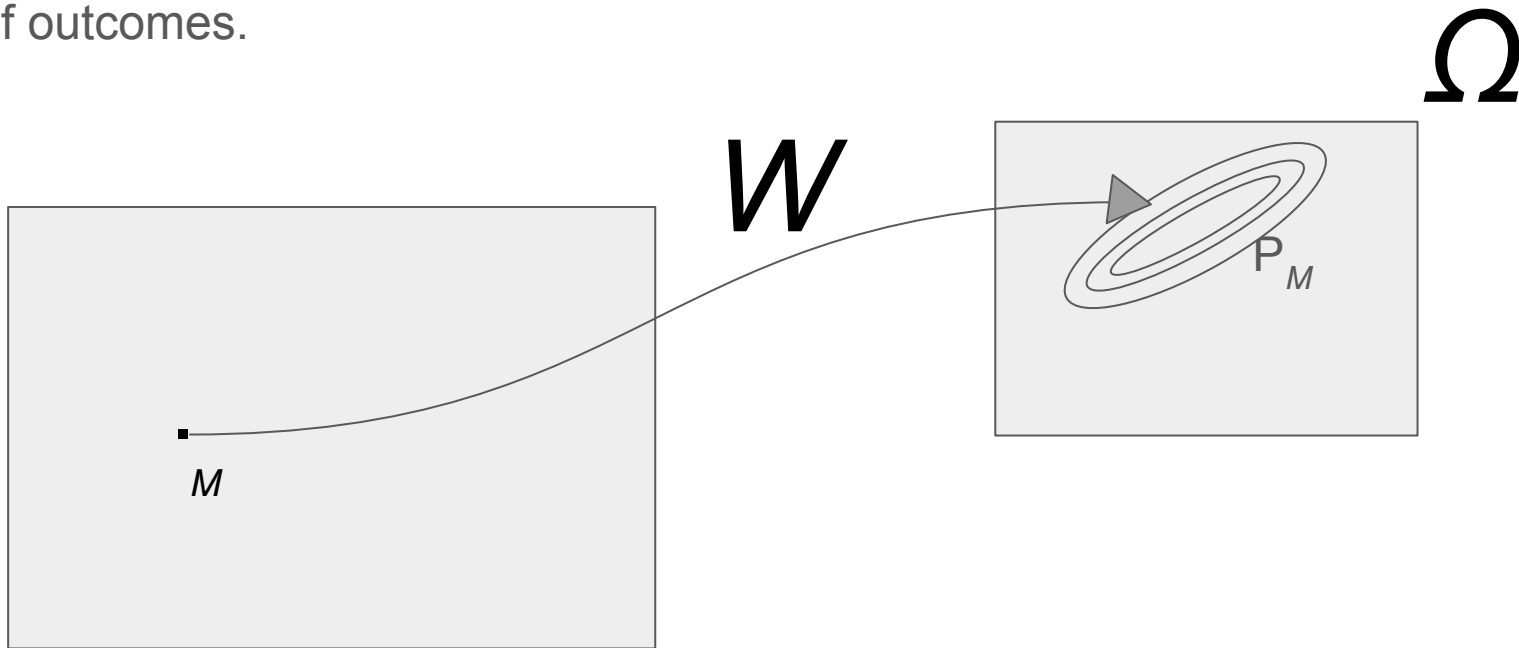
A set  $W$  of **models**, i.e. a set of contextually relevant epistemic possibilities.



$W$

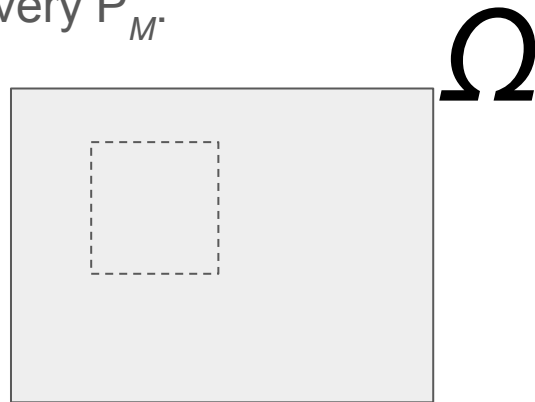
# The Problem Context

Each model  $M$  determines a **unique** probability distribution  $P_M$  over  $\Omega$ , the space of outcomes.



# Weak Topology

- The distributions  $P_n$  converge in the weak topology to  $P$  if for all “feasible” events  $A$ ,  $P_n(A) \rightarrow P(A)$ .
- $A$  is feasible in  $P$  if  $P(\partial A) = 0$ , where  $\partial A$  is the topological boundary of  $A$ .
- We assume that every basic open  $O \subseteq \Omega$  is feasible in every  $P_M$ .



# Statistical Tests

A set of measurable functions  $(T_n)$  is a **test** of  $H$  if each one is a function from samples of size  $n$  to  $\{W, \neg H\}$  and  $T_n^{-1}(\neg H)$  is feasible in every  $P_M$ .

Note: we interpret failure to reject  $H$  as suspension of judgement.

If  $P_{Mn} \Rightarrow P_M$  then, for all tests,  $P_{Mn}(T_n \text{ rejects}) \rightarrow P_M(T_n \text{ rejects})$ .

# Statistical Falsifiability: F2

Say that a hypothesis  $H \subseteq W$  is  **$\alpha$ -falsifiable** iff

- there exists a consistent hypothesis test of  $H$ ;
- with significance level  $\alpha^1$  at every sample size.

Say that a hypothesis  $H$  is **refutable** iff  $H$  is  $\alpha$ -refutable for every  $\alpha > 0$ .

1. A test of  $H$  has significance level  $\alpha$  if the probability of falsely rejecting  $H$  is less than  $\alpha$ .



# Statistical Falsifiability: F2

Say that a hypothesis  $H \subseteq W$  is  **$\alpha$ -falsifiable** iff there is a sequence of tests  $(T_n)$  s.t.

- $P_M(T_n = \text{reject}) \rightarrow 1$  for all  $M \notin H$  as  $n \rightarrow \infty$ ;
- $P_M(T_n = \text{reject}) < \alpha$  for all  $M \in H$ .

Say that a hypothesis  $H$  is **falsifiable** iff  $H$  is  $\alpha$ -refutable for every  $\alpha > 0$ .

# Statistical Falsifiability: F2

**Thm** (Genin and Kelly, 2017). Hypotheses falsifiable in the sense of F2 are exactly the **closed sets** in the **weak topology** on probability measures  $P_M$ .

In other words: falsifiable hypotheses are closed under (1) finite disjunction and (2) arbitrary conjunction.

(But issues of monotonicity are ignored!)

Genin, Konstantin and Kelly, Kevin T. (2017) "The Topology of Statistical Verifiability", Proceedings of TARK XV, Liverpool, pp. 236-50.

# Falsifiability Three (and a half) Ways

**F1.**  $H$  is falsifiable iff there is a monotonic, error-avoiding method  $M$  that falsifies  $H$  in the limit;

**F1.5.**  $H$  is falsifiable iff there is a method  $M$  that falsifies  $H$  in the limit and, for every  $\alpha > 0$ ,  $M$  is  $\alpha$ -error avoiding;

**F2.**  $H$  is falsifiable iff for every  $\alpha > 0$ , there is an  $\alpha$ -error avoiding method  $M$  that falsifies  $H$  in the limit;

**F3.**  $H$  is falsifiable iff or every  $\alpha > 0$ , there is an  $\alpha$ -error avoiding **and  $\alpha$ -monotonic** method  $M$  that falsifies  $H$  in the limit.

# Statistical Falsifiability: F3

Say that a hypothesis  $H \subseteq W$  is  **$\alpha$ -monotonically falsifiable** iff there is a sequence of tests  $(T_n)$  s.t.

- $P_M(T_n = \text{reject}) \rightarrow 1$  for all  $M \notin H$  as  $n \rightarrow \infty$ ;
- $P_M(T_n = \text{reject}) < \alpha$  for all  $M \in H$ ;
- $P_M(T_n = \text{reject}) \downarrow 0$  for all  $M \in H$ ;
- $P_M(T_{n1} = \text{reject}) < P_M(T_{n2} = \text{reject}) + \alpha$  for all  $M \notin H$ .

Say that a hypothesis  $H$  is **monotonically falsifiable** iff  $H$  is  $\alpha$ -monotonically falsifiable for every  $\alpha > 0$ .

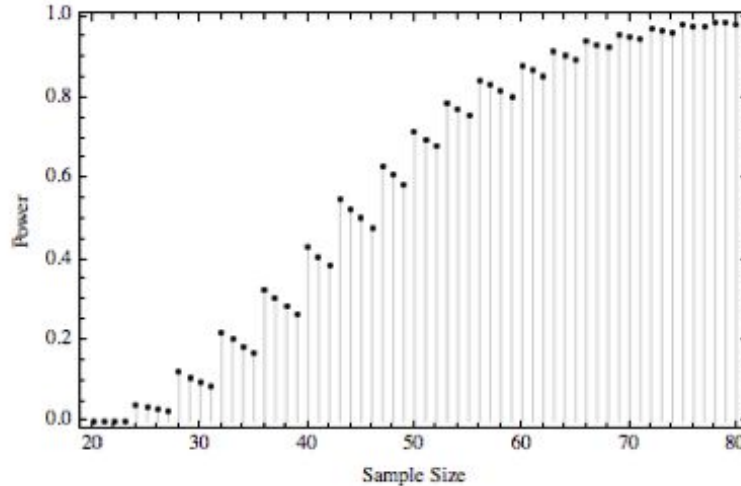
# Statistical Falsifiability: F3

**Thm** (this paper). Hypotheses falsifiable in the sense of **F3** are also exactly the **closed sets** in the **weak topology** on probability measures  $P_M$ .

So insisting on monotonicity does not make any fewer hypotheses falsifiable!

# Statistical Falsifiability: F3

Many standard hypothesis tests are defective!



Chernick, Michael R., and Christine Y. Liu. (2002) "The Saw-Toothed Behavior of Power Versus Sample Size and Software Solutions." *The American Statistician* 56, no. 2: 149–55. <https://doi.org/10.1198/000313002317572835>.

# Why Does this Matter?

- Falsifiability notions F2 and F3 give rise to the exact same falsifiable hypotheses — evidence of a **conceptual robustness**. We can settle questions of falsifiability while remaining ecumenical about what exactly falsifiability means and what events should be taken as falsifying.
- Questions of falsifiability can be settled with **standard** mathematical **techniques** from probability theory and statistics.
- Issues of **monotonicity** are under-developed, especially in discussions of replicable science.

# Why Does this Matter?

Most interesting hypotheses are not falsifiable!

There are exceptions!

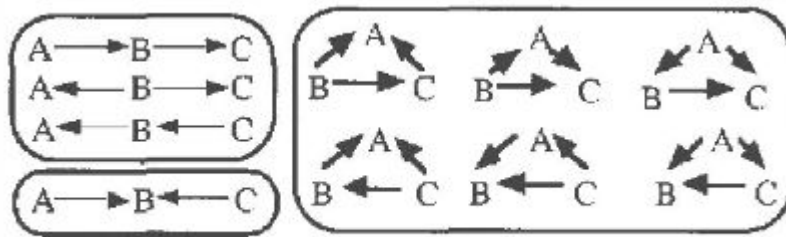


Figure 2: Three Acyclic Markov Equivalence Classes



Genin, Konstantin, and Conor Mayo-Wilson. (2020) "Statistical Decidability in Linear, Non-Gaussian Causal Models." *Proceedings of the Causal Discovery & Causality-Inspired Machine Learning Workshop, NeurIPS, Virtual*.

Genin, Konstantin. (2021) "Statistical Undecidability in Linear, Non-Gaussian Causal Models in the Presence of Latent Confounders." *Advances in Neural Information Processing Systems* 34.

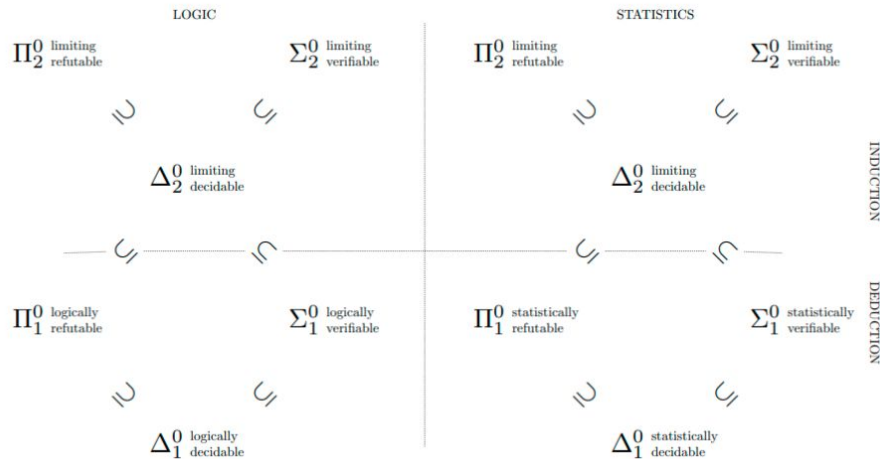
Genin, Konstantin and Conor Mayo-Wilson. (forthcoming) "Success Concepts for Causal Discovery." *Behaviormetrika*.



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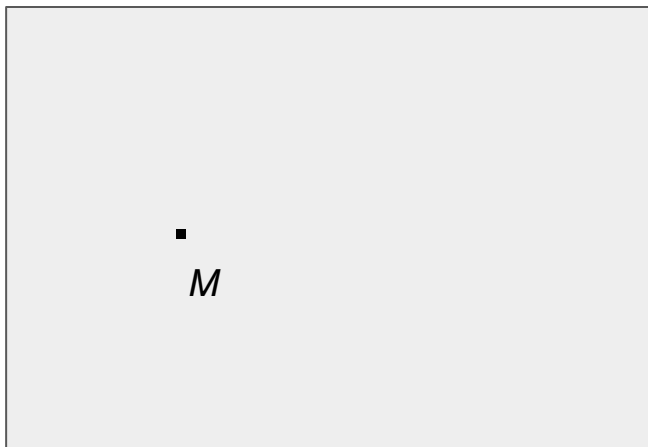
Most interesting hypotheses are not falsifiable!

Mostly True. But falsifiable hypotheses are a fundamental **building block** of higher-complexity hypotheses.



# The Problem Context

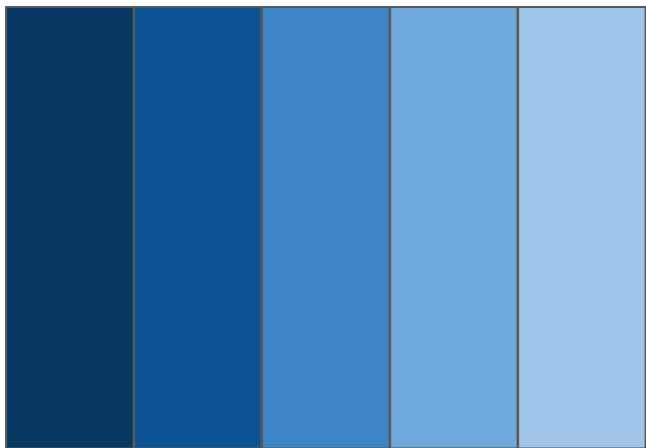
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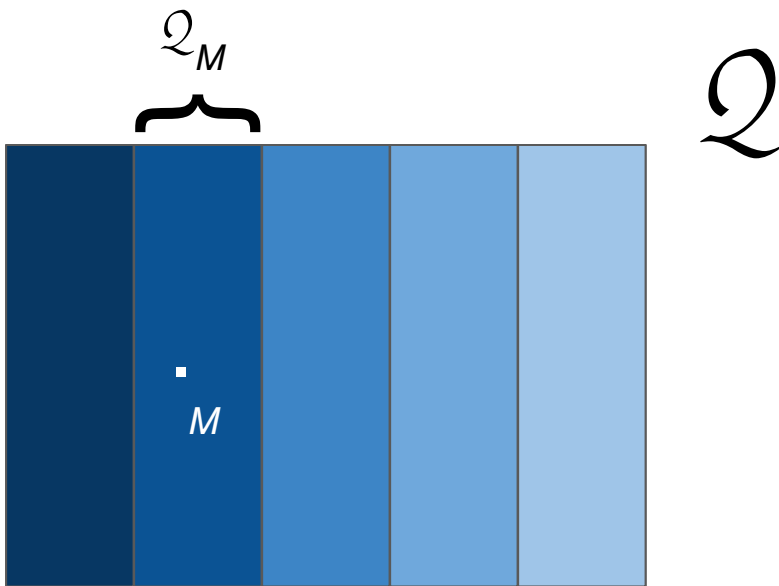
A **question**  $\mathcal{Q}$ , partitioning  $W$  into a countable set of **answers**.



$\mathcal{Q}$

# Statistical Questions

A **question**  $\mathcal{Q}$ , partitioning  $W$  into a countable set of **answers**.



# Statistical Solutions

A method  $(L_n)$  is a **solution** to  $\mathcal{Q}$  iff for all  $M$  in  $W$ ,

- $P_M(L_n = \mathcal{Q}_M) \rightarrow 1$  as  $n \rightarrow \infty$ .

A question  $\mathcal{Q}$  is **solvable** iff it has a solution.

# Solvable Problems

**Thm.** A question  $\mathcal{Q}$  is **solvable** iff each answer  $A$  in  $\mathcal{Q}$  is a countable union of statistically falsifiable hypotheses.

Dembo, A., & Peres, Y. (1994). A topological criterion for hypothesis testing. *The Annals of Statistics*, 106-117.

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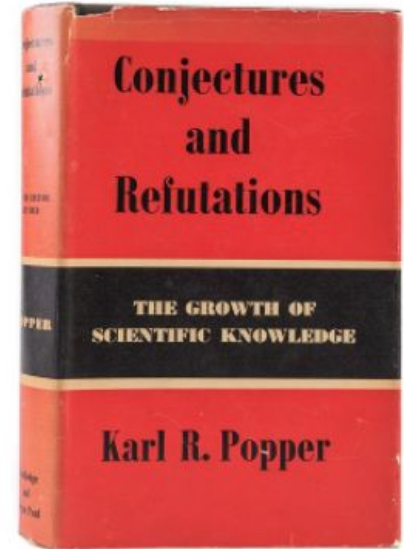
# Why Does this Matter?

**The Problem of Progress:** Why is scientific method conducive to scientific progress?

**The Problem of Ockham's Razor:** How is a consistent preference for simple theories conducive to scientific progress?

# The Problem of Progress

**Popper's Story:** Science progresses through a series of simple (highly testable) conjectures, followed by dogged attempts at refutation.

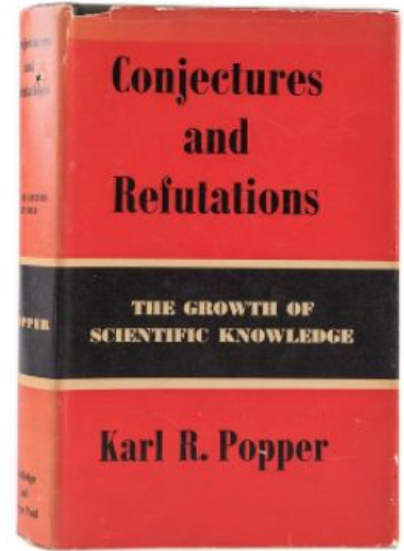




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But why think this is anything more than a series of **bold mistakes**, yielding to new, and **bolder**, mistakes?



# The Problem of Progress

**Lakatos Objects:** Popper “offers a methodology without an **epistemology** or a **learning theory**, and confesses explicitly that his methodology may lead us epistemologically astray and, implicitly, that *ad hoc* stratagems might lead us to Truth.”

*The Role of Crucial Experiments in Science* (1971).

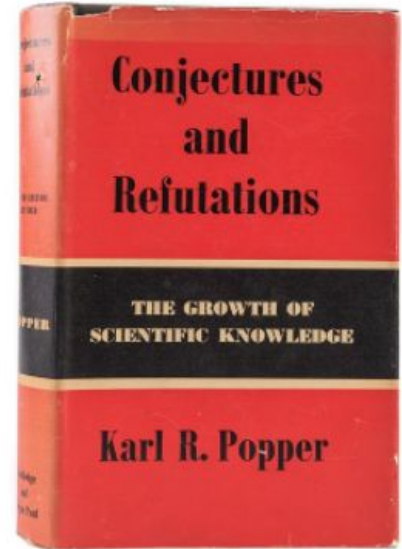


# The Problem of Progress

Popper hoped to show that the critical method leads to theories of **increasing truthlikeness**.

$T_1$  is at least as verisimilar as  $T_2$  iff

$T_1$  has at least as many true consequences as, and no more false consequences than,  $T_2$ .



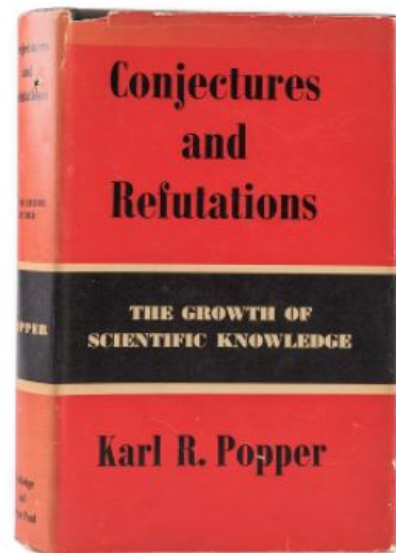
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Popper hoped to show that the critical method leads to theories of **increasing truthlikeness**.

$T_1$  is at least as verisimilar as  $T_2$  iff

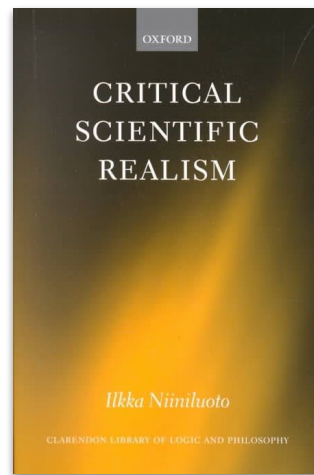
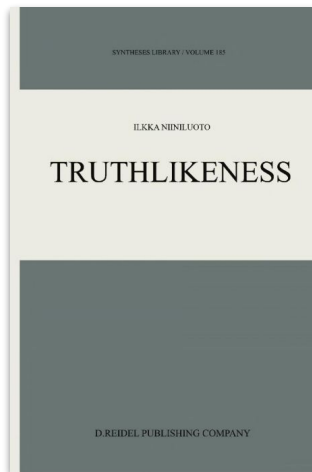
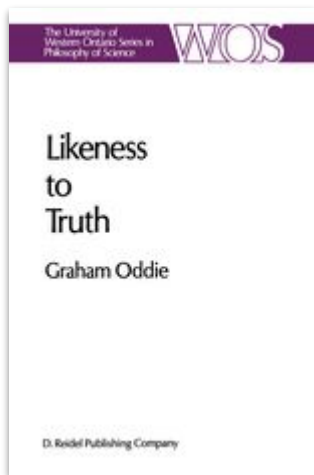
$T_1$  has at least as many true consequences as, and no more false consequences than,  $T_2$ .

But Popper's truthlikeness (verisimilitude) was famously **trivialized** by Pavel Tichy (1974) and David Miller (1974).



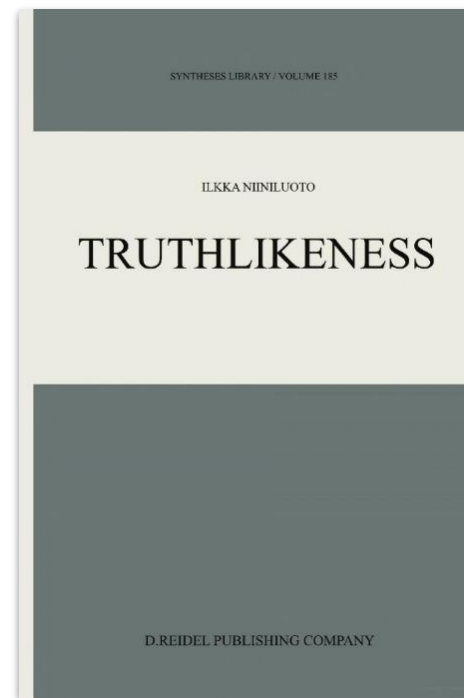
# The Problem of Progress

Oddie (1986) and Niiniluoto (1987, 1999) make more sophisticated iterations of truthlikeness.



# The Problem of Progress

“... the problem of estimating verisimilitude is **neither more nor less difficult** than the traditional **problem of induction**” (1987).



# Progressive Methods

A **method** for answering a scientific question is **progressive** iff

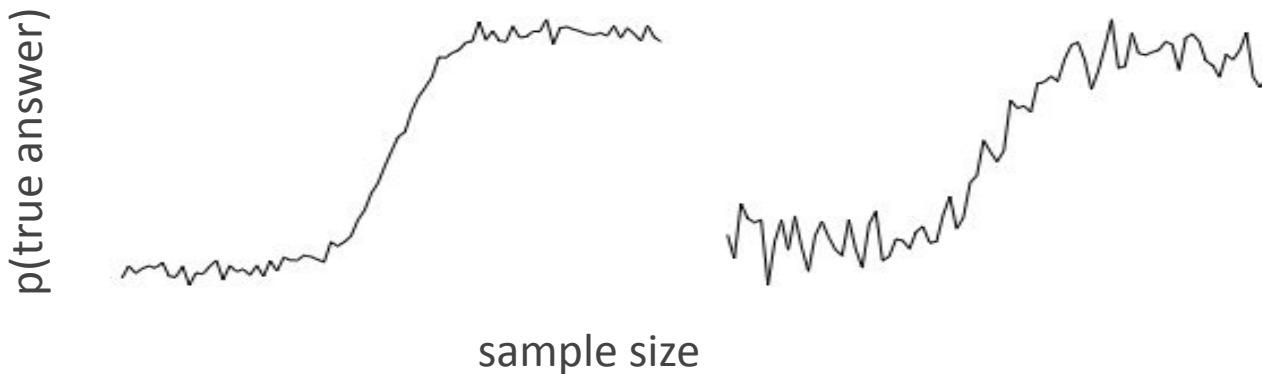
- the **chance** that it outputs the **true** answer is **strictly increasing** with sample size.

That notion makes sense, even if it doesn't make sense to ask which of two false theories is closer to the truth!

# Progressive Methods

A **method** for answering a scientific question is  $\alpha$ -**progressive** iff

- the **chance** that it outputs the **true** answer **never drops** by more than  $\alpha$ .





# Progressive Solutions

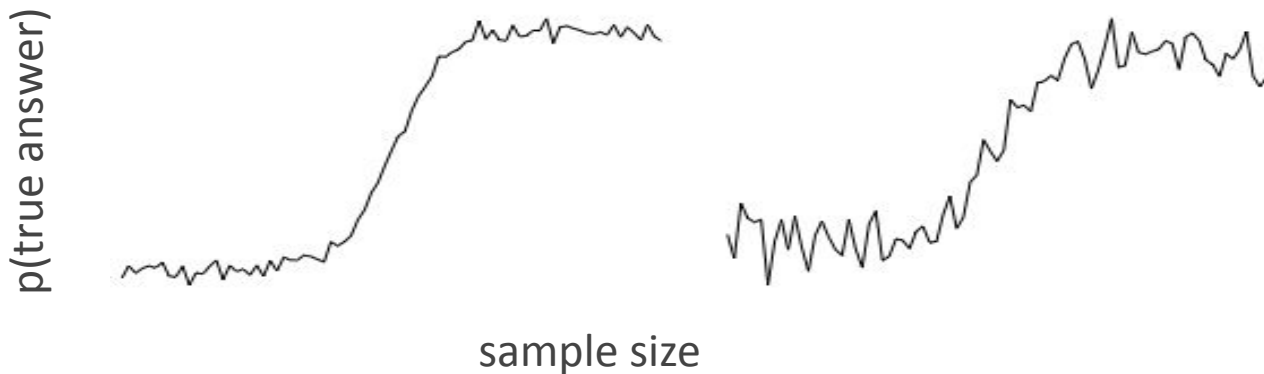
A solution to  $\mathcal{Q}(L_n)$  is **progressive** iff for all  $M$  in  $W$  and  $n_1 < n_2$ ,

- $P_M(L_{n_1} = \mathcal{Q}_M) < P_M(L_{n_2} = \mathcal{Q}_M)$ .

# $\alpha$ -Progressive Solutions

A solution to  $\mathcal{Q}(L_n)$  is  $\alpha$ -**progressive** iff for all  $M$  in  $W$  and  $n_1 < n_2$ ,

- $P_M(L_{n_1} = \mathcal{Q}_M) < P_M(L_{n_2} = \mathcal{Q}_M) + \alpha$ .



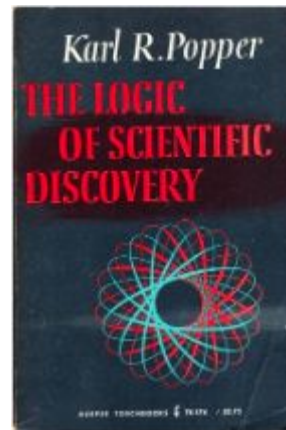
Problem  $\mathcal{Q}$  is **progressively solvable** iff it has an  $\alpha$ -progressive solution for all  $\alpha > 0$ .

# Statistical Simplicity.

For  $A, B$  sets of models, say that  $A$  is **as simple as**  $B$  iff

- All refutable consequences of  $B$  are refutable consequences of  $A$ .
- Topologically:  $A \subseteq \text{cl}(B)$ .

*But this confounds logical strength and simplicity!*



# Statistical Simplicity.

For  $A, B$  **disjoint** sets of models, say that  $A$  is **as simple as**  $B$  iff

- All refutable consequences of  $B$  are refutable consequences of  $A$ .
- $A, B$  disjoint and  $A \subseteq \text{cl}(B)$ .

# Statistical Simplicity.

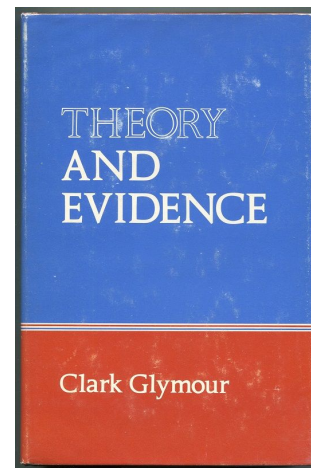
For  $A, B$  **disjoint** sets of models, say that  $A$  is **as simple as**  $B$  iff

- All refutable consequences of  $B$  are refutable consequences of  $A$ .
- $A, B$  disjoint and  $A \subseteq \text{cl}(B)$ .

*Tack-on problem!*

LIN is simpler than QUAD.

LIN or the cat is on the mat is **not** simpler than QUAD.



# Statistical Simplicity.

For  $A, B$  disjoint sets of models, say that  $A$  is **as simple as**  $B$  iff

- All refutable consequences of  $B$  are **consistent** with  $A$ .
- $A \cap \text{cl}(B) \neq \emptyset$ .

# Progressive Solutions

**Theorem** (Genin, 2018). If the answers to  $\mathcal{Q}$  can be enumerated in agreement with simplicity, then it is progressively solvable.

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If  $A_1, A_2, A_3 \dots$  is a enumeration of the answers agreeing with simplicity, a progressive method can be constructed by attempting to falsify larger initial segments of answers.



# Progressive Solutions

**Theorem** (Genin, 2018). If the answers to  $\mathcal{Q}$  can be enumerated in agreement with simplicity, then it is progressively solvable.

If  $A_1, A_2, A_3 \dots$  is an enumeration of the answers agreeing with simplicity, a progressive method can be constructed by attempting to falsify larger initial segments of answers.

But the constituent falsifying methods must be **sufficiently monotonic!**

# Ockham's $\alpha$ -Razor

**Defn.** Solution  $(L_n)$  satisfies Ockham's  $\alpha$ -razor iff (in each  $M$ ) the chance of conjecturing an answer more complex than  $\mathcal{Q}_M$  is less than  $\alpha$ .



# Ockham's $\alpha$ -Razor

**Defn.** Solution  $(L_n)$  satisfies Ockham's  $\alpha$ -razor iff

If  $\mathcal{Q}_M$  is simpler than  $A \in \mathcal{Q}$ , then  $P_M(L_n = A) < \alpha$ .



# Progress and Simplicity

**Theorem** (Genin, 2018). Every  $\alpha$ -progressive solution to  $\mathcal{Q}$  satisfies Ockham's  $\alpha$ -razor.



# Progress and Simplicity

**Theorem** (Genin, 2018). Every  $\alpha$ -progressive solution to  $\mathcal{Q}$  satisfies Ockham's  $\alpha$ -razor.

A **non-circular, epistemic** justification of Ockham's razor in statistical science.



# Progress and Simplicity

**Theorem** (Genin, 2018). Every  $\alpha$ -progressive solution to  $\mathcal{Q}$  satisfies Ockham's  $\alpha$ -razor.

Violating Ockham's razor means **designing in** a tendency to **fail** to replicate **true** results.



# Progress and Simplicity

**Theorem** (Genin, 2018). Every  $\alpha$ -progressive solution to  $\mathcal{Q}$  satisfies Ockham's  $\alpha$ -razor.

A carefully calibrated Popperian methodology can ensure  
That the degree of backsliding is **arbitrarily** low.



# Synchronic Justification

Traditionally, epistemic justification has been **synchronically** conceived:

“Justifying an epistemic principle requires answering an epistemic question: why are simpler theories more likely to be true?”



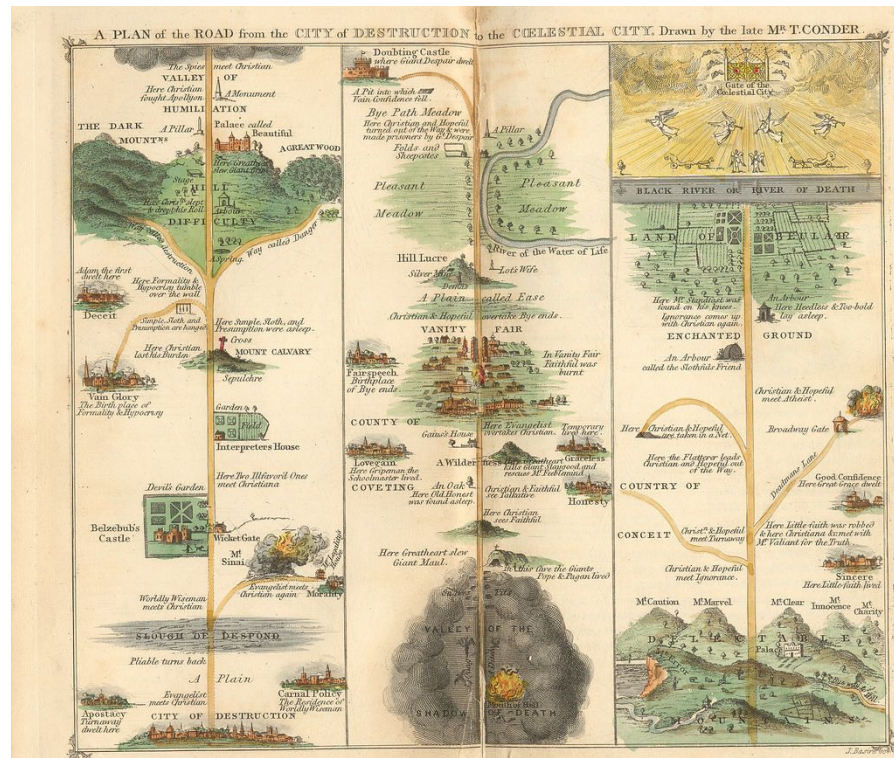
# Synchronic Justification

Demanding synchronic justification leads to **despair**:

“[N]o one has shown that any of these rules is more likely to pick out true theories than false ones. It follows that none of these rules is epistemic in character.”

# Diachronic Justification

Going diachronic allows us to demonstrate that a systematic preference for simple theories is a necessary condition for a diachronic notion of **progress**.



# Thank You!

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