



# Simplicity and Scientific Progress

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# The **Synchronic** and **Diachronic** Schools

**Synchronic School:** focused on the finished products of science, esp. characterizing which beliefs (or systems of belief) constitute **rational** responses to evidence.

**Diachronic School:** characterize which methods are conducive to scientific progress.

Ilkka Niiniluoto, *Scientific Progress* (2015)

# Diachronic School

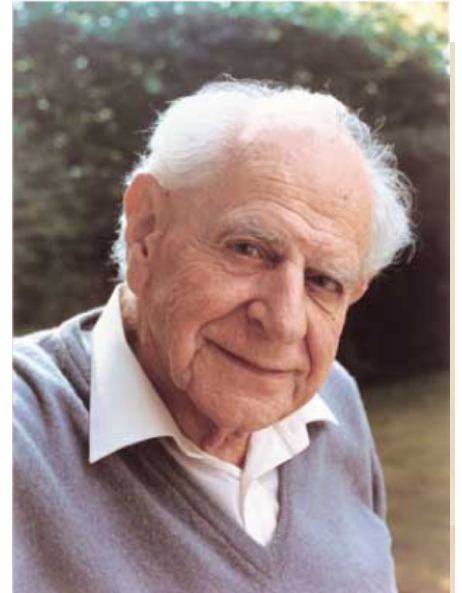
“... progress necessarily involves the idea of a **process through time**. Rationality, on the other hand, has tended to be viewed as an **atemporal** concept ... most writers see progress as nothing more than the temporal projection of a series of individual rational choices .... we may be able to learn something **by inverting the presumed dependence of progress on rationality.**”



Laudan, *Progress and its Problems* (1978).

# Popper's Critical Rationalism

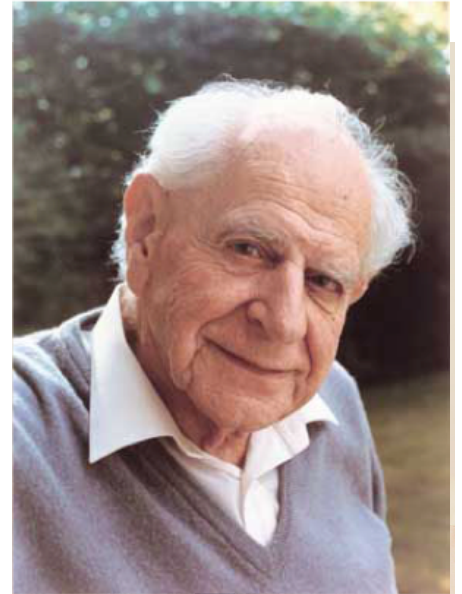
**Popper:** Science progresses through a series of highly testable conjectures, followed by dogged attempts at refutation.



# Popper's Critical Rationalism

**Popper:** Science progresses through a series of highly testable conjectures, followed by dogged attempts at refutation.

But why think this is anything more than a series of **bold mistakes**, yielding to new, and bolder, mistakes?



# Lakatos Objects

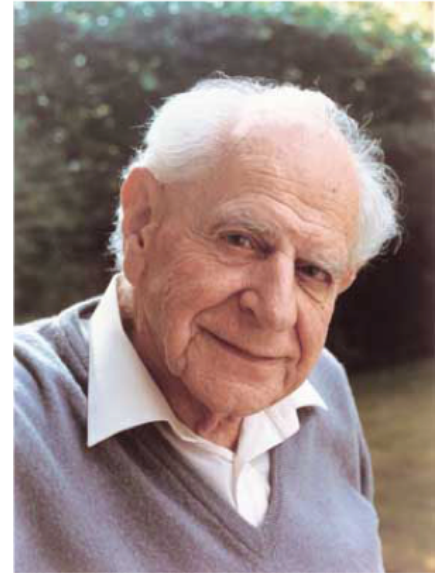
Popper “offers a methodology without an epistemology or a learning theory, and confesses explicitly that his methodology may lead us epistemologically astray, and implicitly, that *ad hoc* stratagems might lead us to Truth.”

Imre Lakatos, *The Role of Crucial Experiments in Science* (1971).



# Truthlikeness

Popper developed a theory of verisimilitude, hoping to show that the process of conjectures and refutations leads to theories of **increasing truthlikeness** (1963, 1970).



Popper's idea was famously **trivialized** (independently) by Pavel Tichy and David Miller (1974). On Popper's account, no false theory is more truthlike than any other!

# Truthlikeness Redux

Oddie (1986) and Niiniluoto (1987, 1999) make more sophisticated attempts at a definition of truthlikeness.



# Truthlikeness Redux

But there is no demonstration that any method is guaranteed to produce increasingly truthlike theories!

# Truthlikeness Redux

“appraisals of the relative distances from the truth presuppose that an epistemic probability distribution . . . is available. In this sense ... the problem of estimating verisimilitude is neither more nor less difficult than the traditional problem of induction.”



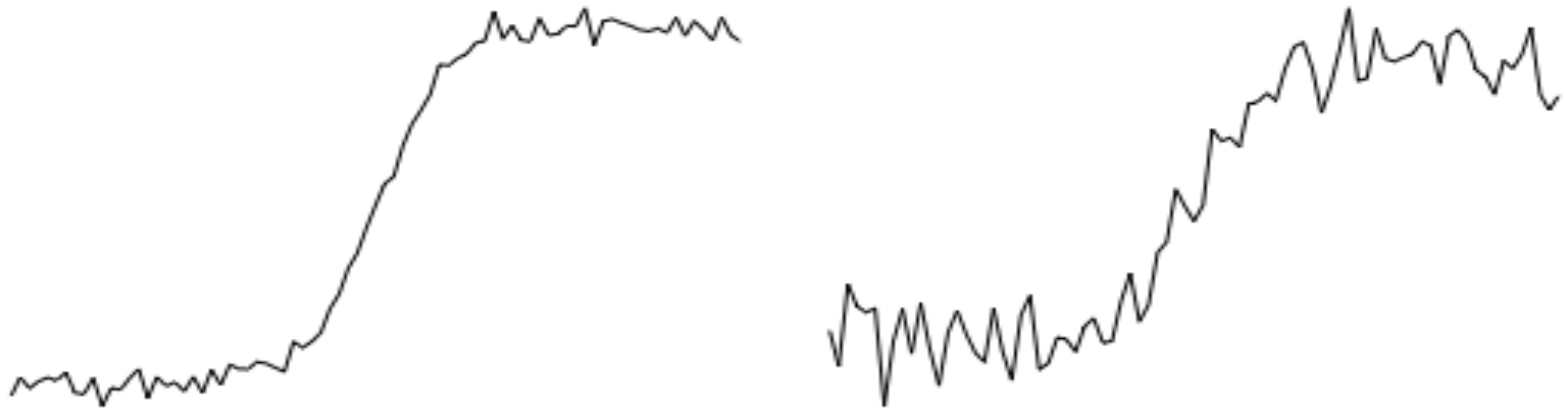
Ilkka Niiniluoto, *Truthlikeness* (1987).

# Progressive Methods

- Say that a method for answering a question is **progressive** if the **chance** that it outputs the **true answer** is strictly **increasing** with sample size.
- That notion makes sense, even if it does not make sense to ask which of two false theories is closer to the truth!

# Progressive Methods

- A method is  $\alpha$ -progressive if the chance that it outputs the true answer **never decreases** by more than  $\alpha$ .



# Progressive Methods

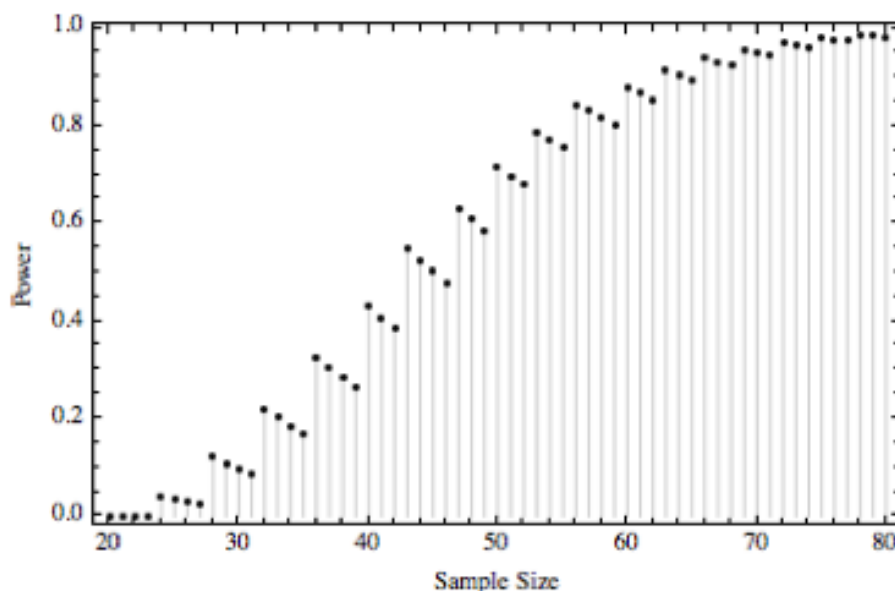
Researchers propose recruiting 100 patients to investigate whether a new drug is better at treating migraine than placebo. In their grant, they analyze their statistical method and conclude the following: if the new drug is significantly better than placebo, the chance that their method detects the improvement is greater than 50%. The funding agency is satisfied. Soon after, the researchers publish a paper claiming to have discovered a promising new treatment!

# Progressive Methods

Now, suppose that a replication study is proposed with 150 patients. However, the ex ante analysis reveals that the objective chance of detecting an improvement over placebo, if one exists, has decreased to 40%. The **chance of replicating successfully** has gone **down**, even though the first study may well be correct, and yet the investigators propose performing a larger study!

# Progressive Methods

Surprisingly, many textbook methods in frequent hypothesis testing exhibit this perverse behavior.



Chernick and Liu, *The Saw-toothed behavior of power vs. sample size and software solutions.* (2012)

# Progressive Methods

**Theorem (Genin):** For typical problems, there exists an  $\alpha$ -progressive method for every  $\alpha > 0$ .



# A Vindication of Neo-Popperian Method

**Theorem (Genin):** All progressive methods must systematically prefer simpler (more falsifiable) theories.

# The Plan

1. Prove this result in the simplified setting of **propositional** information.
2. Port this result to the setting of **statistical** information.

# The Topological Bridge

- Start with **logical** insights.
- Allow methods a small chance  $\alpha$  of error.
- Obtain corresponding **statistical** insights



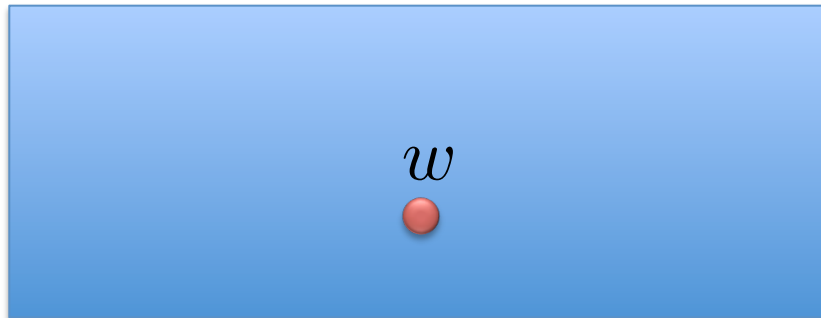
# The Topology of Information

I ● topology



# Possible Worlds

$W$



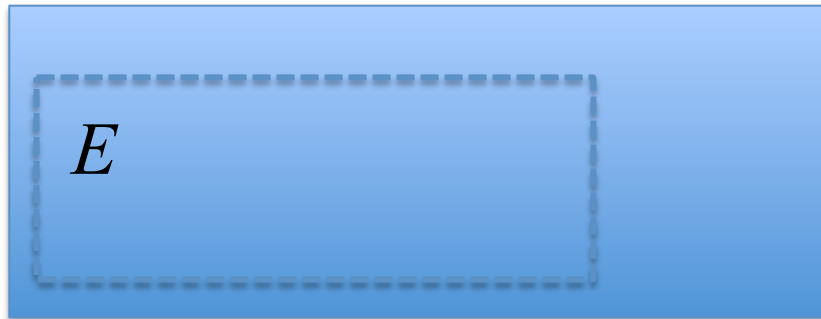
$w$



# Propositional Information State

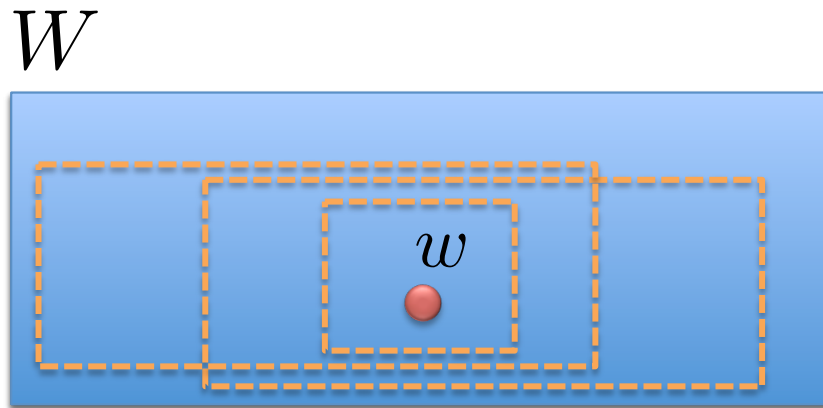
The **logically strongest** proposition you are informed of.

$W$



# Propositional Information State

- $\mathcal{I}$  is the set of **all** possible information states.
- $\mathcal{I}(w)$  is the set of all information states **true** in  $w$ .
- $\mathcal{I}(w \mid E) = \{ F \text{ in } \mathcal{I}(w) : F \subseteq E \}$

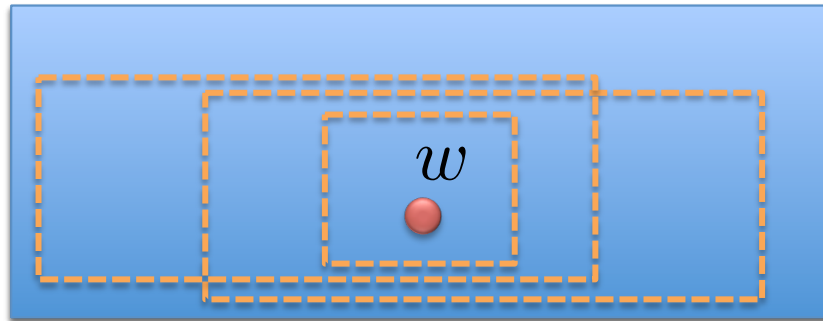


# Propositional Information State

**Intended Interpretation:**  $E$  is in  $\mathcal{I}(w)$  iff

a diligent inquirer in  $w$  will eventually be afforded information at least as strong as  $E$ .

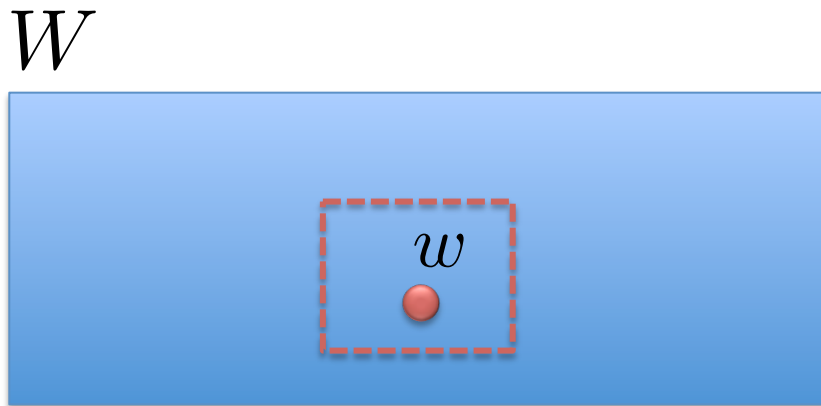
$W$





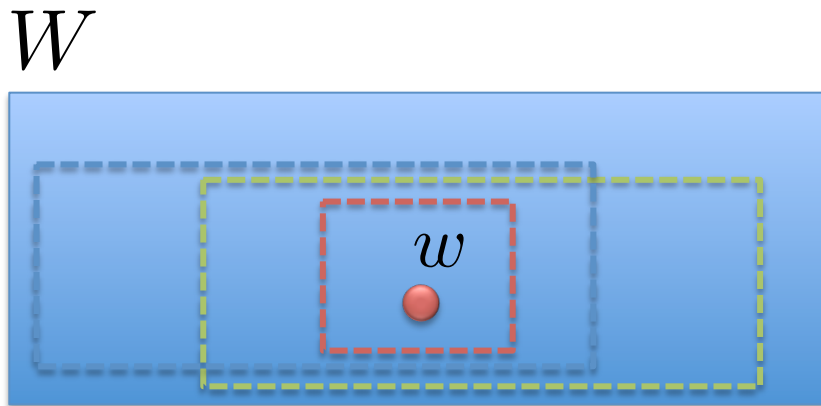
# Three Axioms

1. **Some** information state is true in  $w$ .



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2. Each pair of information states **true** in  $w$  is **entailed** by an information state **true** in  $w$ .



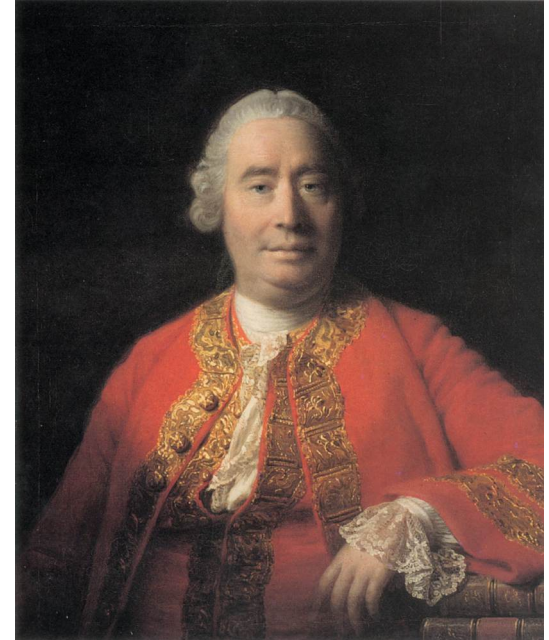
# Three Axioms

1. Some information state is true in  $w$ .
2. Each pair of information states true in  $w$  is entailed by an information state true in  $w$ .
3. There are at most **countably many** information states.

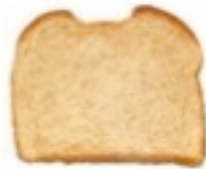
# Hume's Problem

“The bread, which I formerly ate, nourished me ... but does it follow, that **other** bread must **also** nourish me at another time ... ? The consequence seems nowise necessary.”

Hume, *Enquiry*.



# Hume's Problem, Topologized.



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# Hume's Problem, Topologized.

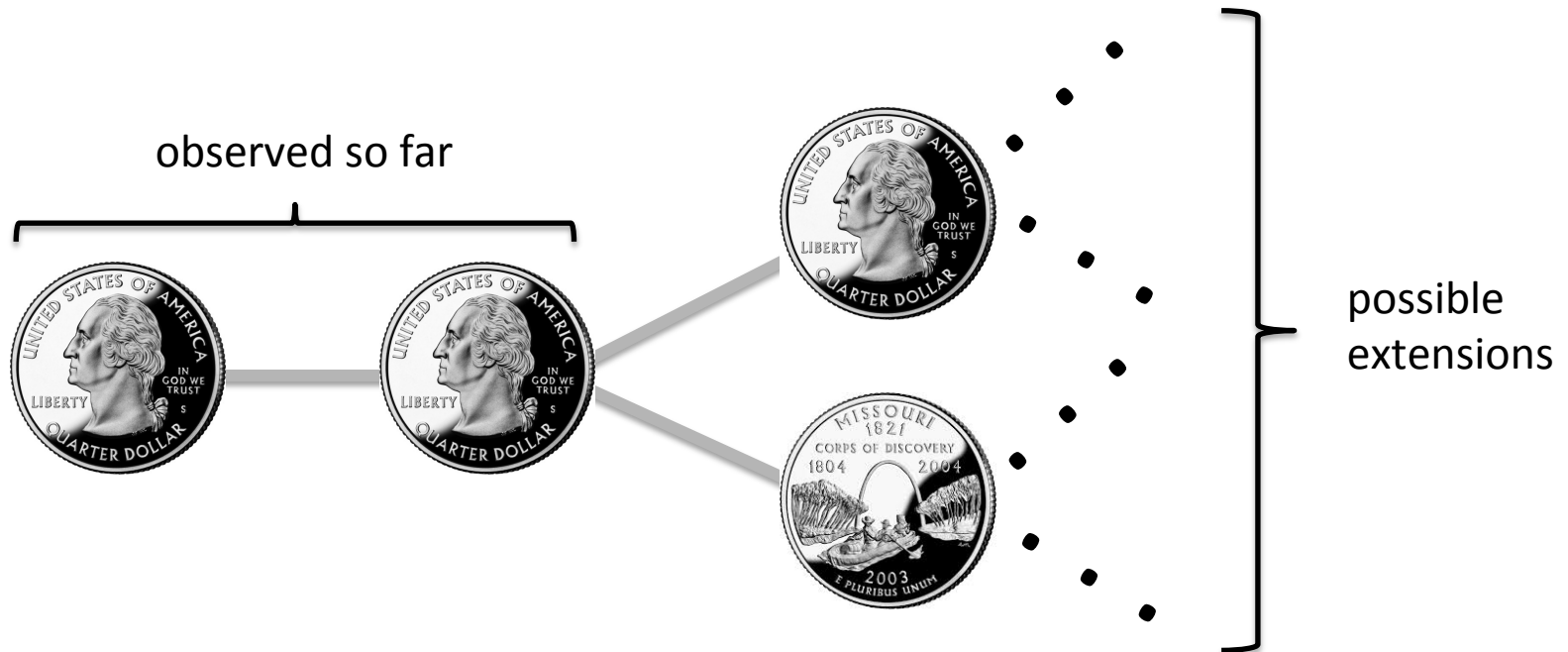




# Example: Sequential Binary Experiment

**Worlds** = infinite sequences of coin flips.

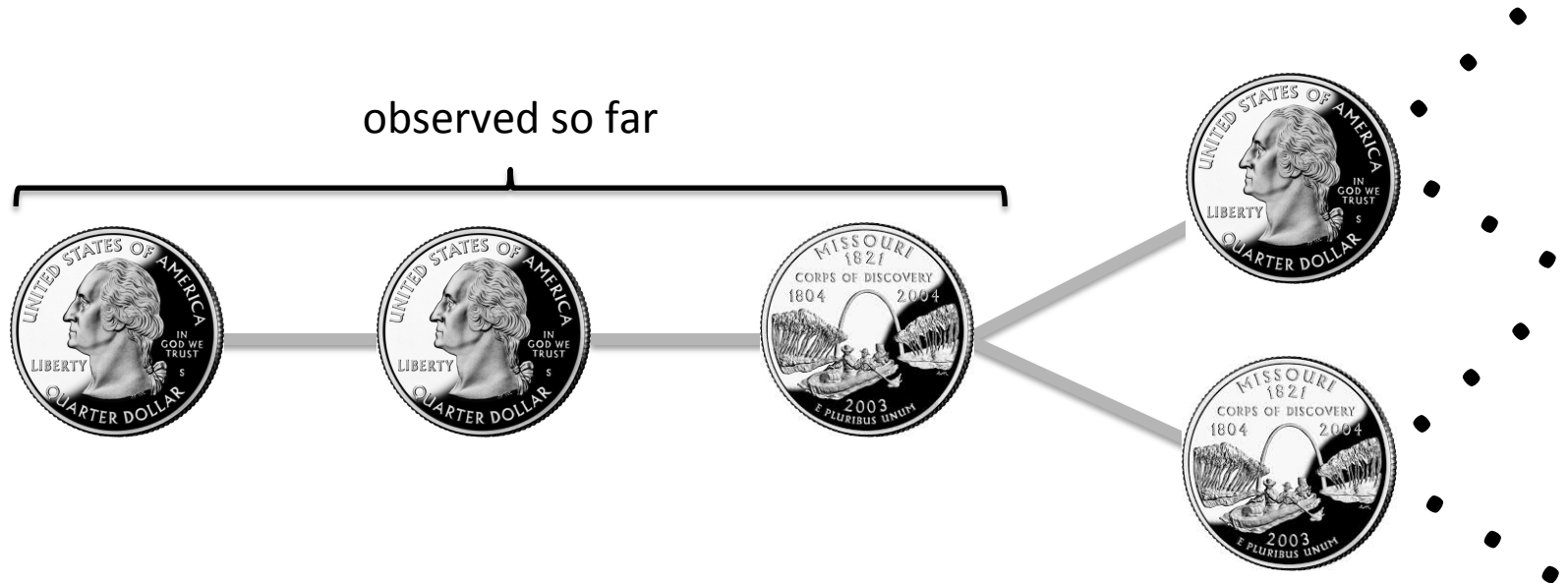
**Evidential states** = cones of possible extensions of finite sequences:



# Example: Sequential Binary Experiment

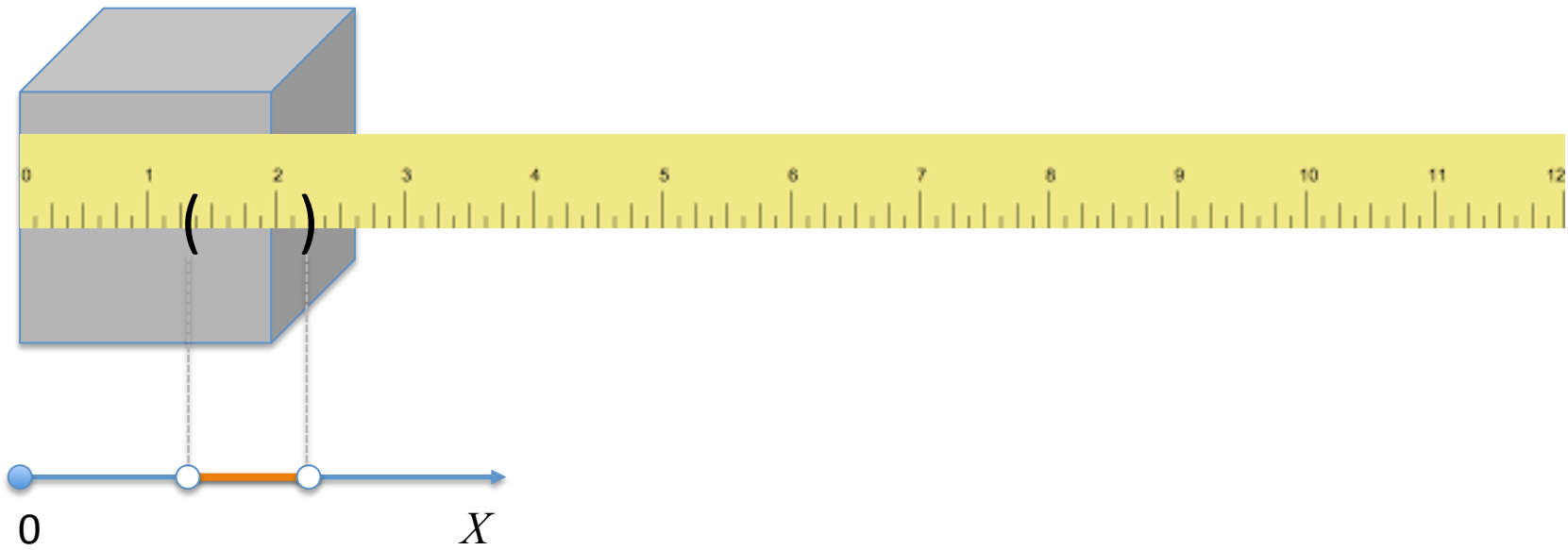
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**Evidential states** = cones of possible extensions of finite sequences:



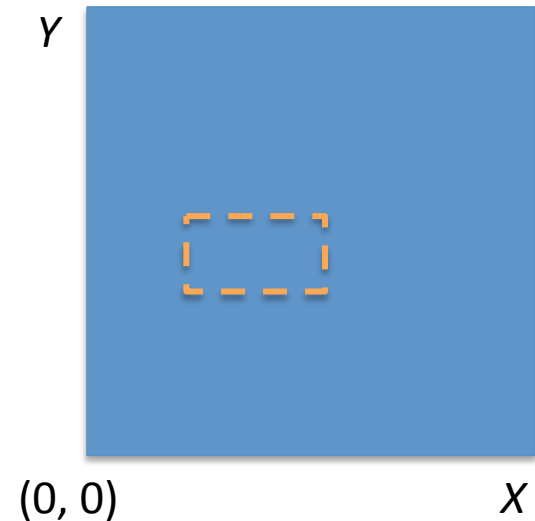
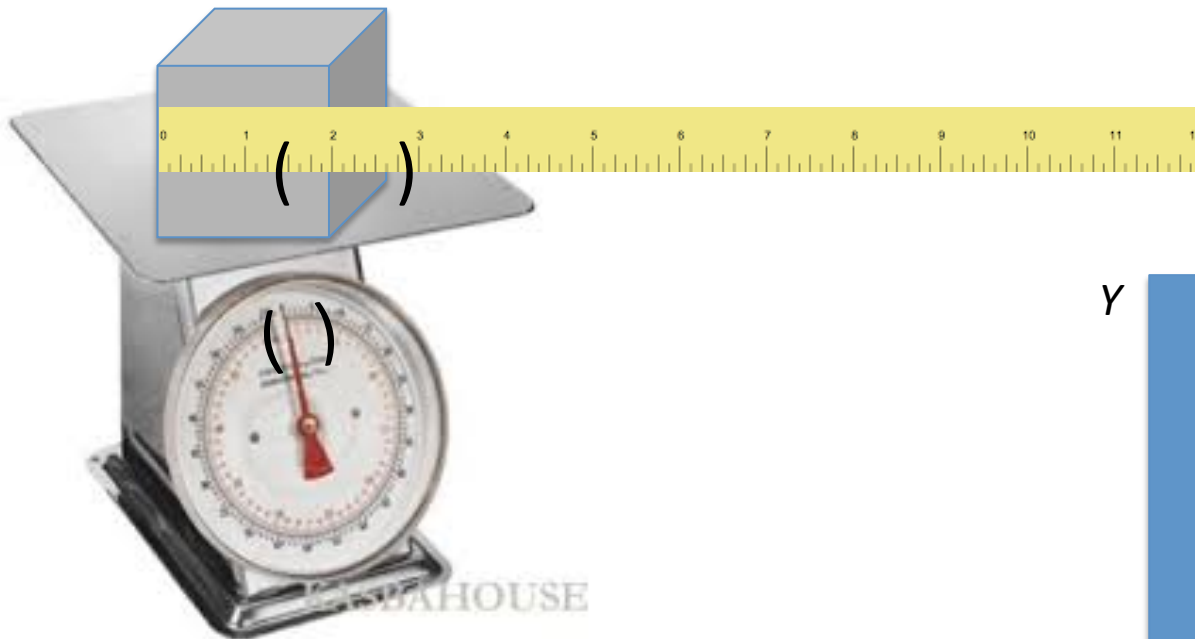
# Example: Measurement of $X$

- **Worlds** = real numbers.
- **Information states** = open intervals.



# Example: Joint Measurement

- **Worlds** = points in real plane.
- **Information states** = open rectangles.



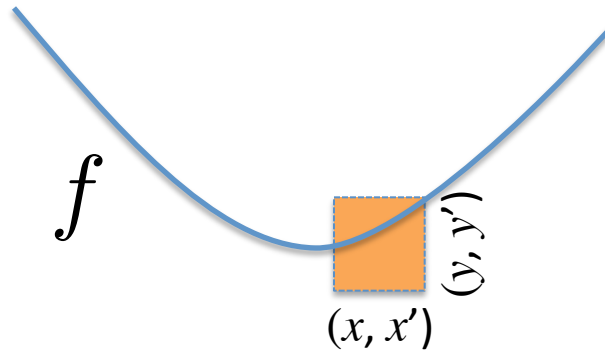
# Example: Functions

- **Worlds** = functions  $f : \mathbb{R} \rightarrow \mathbb{R}$ .



# Example: Functions

- An **observation** is a joint measurement.



# Example: Functions

- The **information state** is the set of all worlds that **touch** each observation.

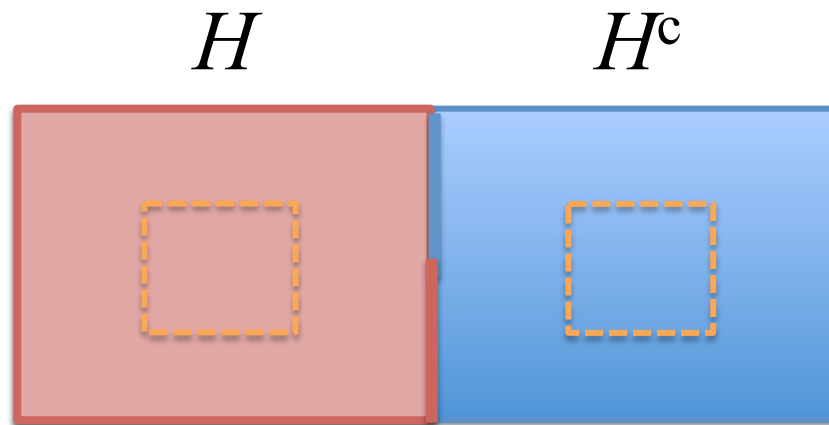


# Deductive Verification and Refutation

$H$  is **verified** by  $E$  iff  $E \subseteq H$ .

$H$  is **refuted** by  $E$  iff  $E \subseteq H^c$ .

$H$  is **decided** by  $E$  iff  $H$  is either **verified** or **refuted** by  $E$ .



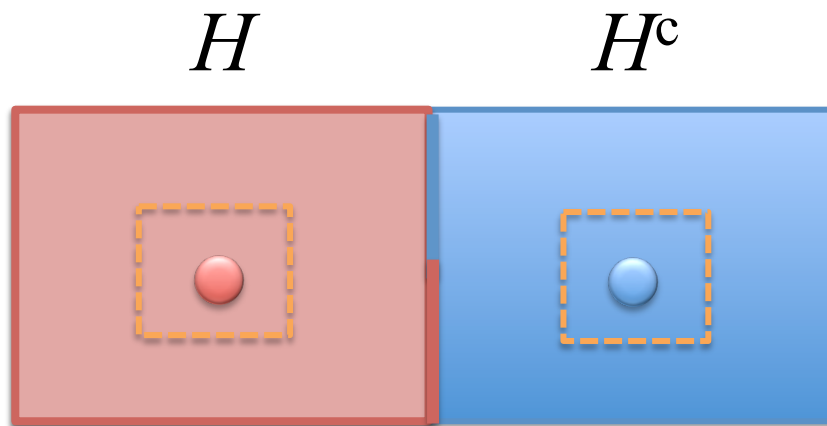


# Will be Verified

$w$  is an **interior point** of  $H$  iff

iff  $H$  **will be verified** in  $w$ ;

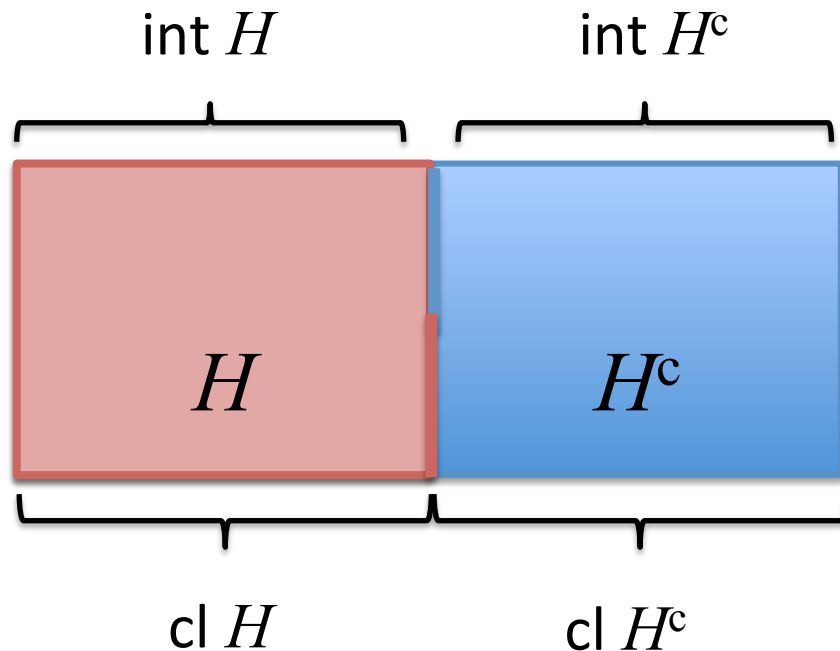
iff there is  $E$  in  $\mathcal{I}(w)$  s.t.  $H$  is **verified** by  $E$ .



# Topological Operators as Modal Operators

**int  $H$**  := the proposition that  $H$  **will be verified**.

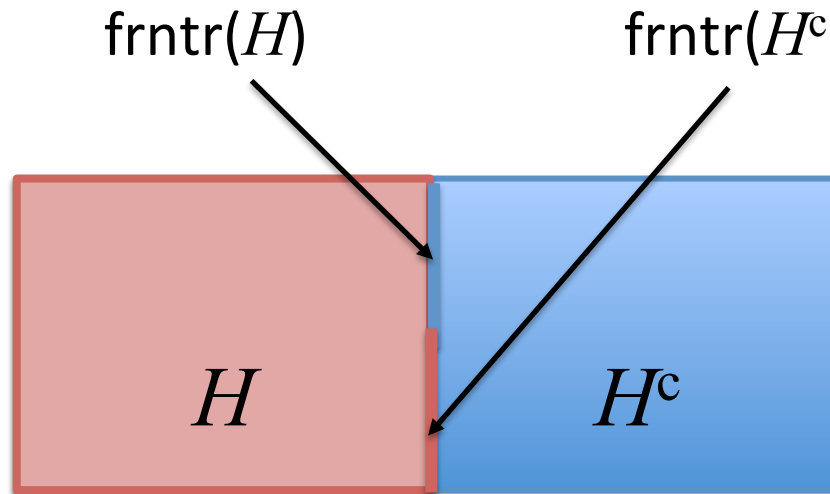
**cl  $H$**  := the proposition that  $H$  **will never be refuted**.



# Topological Operators

**frntr**  $H$  := the proposition that  $H$  is **false** but will never be **refuted**.

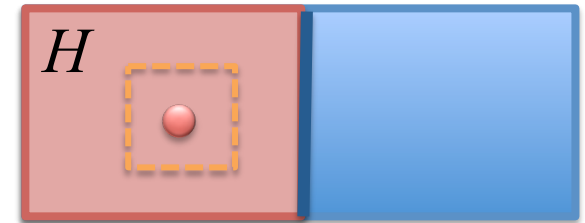
**frntr**  $H^c$  := the proposition that  $H$  is **true** but will never be **verified**.



# Verifiability, Refutability, Decidability

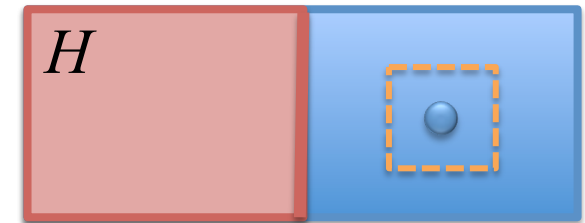
$H$  is **verifiable (open)** iff  $H \subseteq \text{int}(H)$ .

i.e., iff  $H$  will be **verified** however  $H$  is **true**.

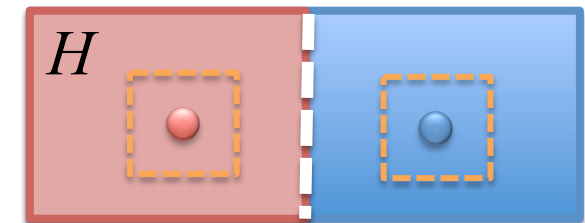


$H$  is **refutable (closed)** iff  $\text{cl}(H) \subseteq H$ .

i.e., iff  $H$  will be **refuted** however  $H$  is **false**.

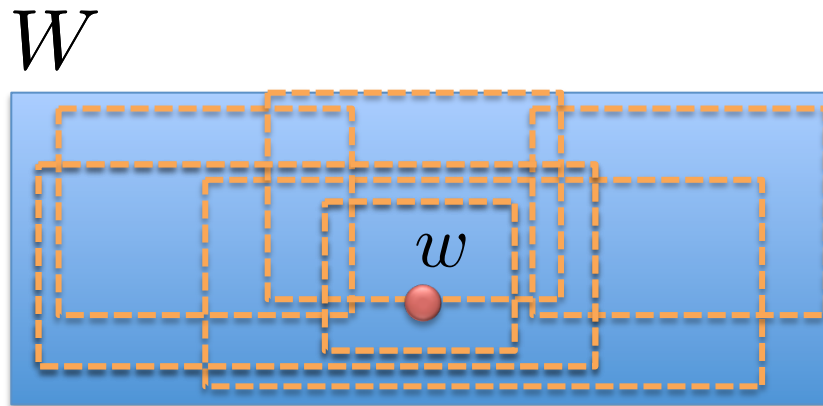


$H$  is **decidable (clopen)** iff  $H$  is both verifiable and refutable.



# The **Topology** of Information

- A **topology** on  $W$  is determined by its **open** (verifiable) propositions.
- Every verifiable proposition is a **disjunction** of information states in  $\mathcal{I}$ .



# Interior

**int**  $H$  = the proposition that  $H$  **will be verified**.



$$\text{Int} \{ \text{skull} \} = \{ \text{skull} \}$$

$$\text{Int} \{ \text{toast} \} = \emptyset$$

# Open = Verifiable

$H$  is **open (verifiable)** iff  $H$  entails **int**  $H$ .



$$\text{Int } \{\text{skull}\} = \{\text{skull}\}$$

$$\text{Int } \{\text{toast}\} = \emptyset$$

# Closure

$\text{cl } H$  = the proposition that  $H$  **will never be refuted**.



$$\text{Cl} \{ \text{skull} \} = \{ \text{toast}, \text{skull} \}$$

$$\text{Cl} \{ \text{toast} \} = \{ \text{toast} \}$$



# Closed = Refutable

$H$  is **closed (refutable)** iff  $\mathbf{cl} H$  entails  $H$ .



$$\mathbf{cl} \{ \text{👤} \} = \{ \text{🍞}, \text{👤} \}$$

$$\mathbf{cl} \{ \text{🍞} \} = \{ \text{🍞} \}$$

# Frontier

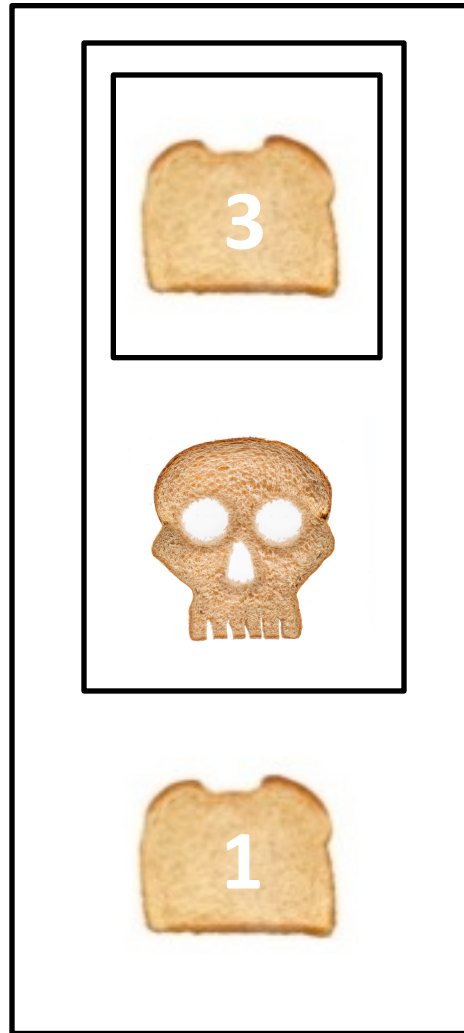
**frntr**  $H = H$  is false, but will never be refuted.



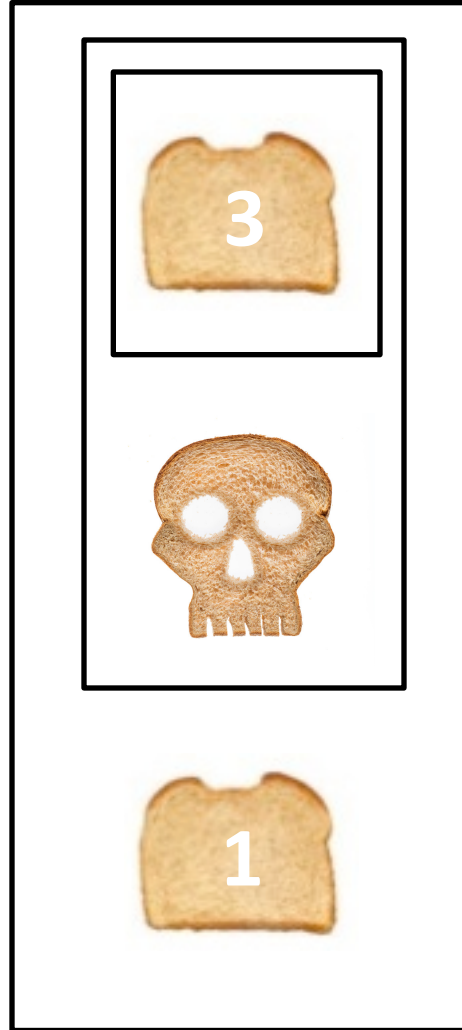
$$\text{Frntr} \{ \text{skull} \} = \{ \text{cookie} \}$$

$$\text{Frntr} \{ \text{cookie} \} = \emptyset$$

# Hume's Problem, Enhanced.

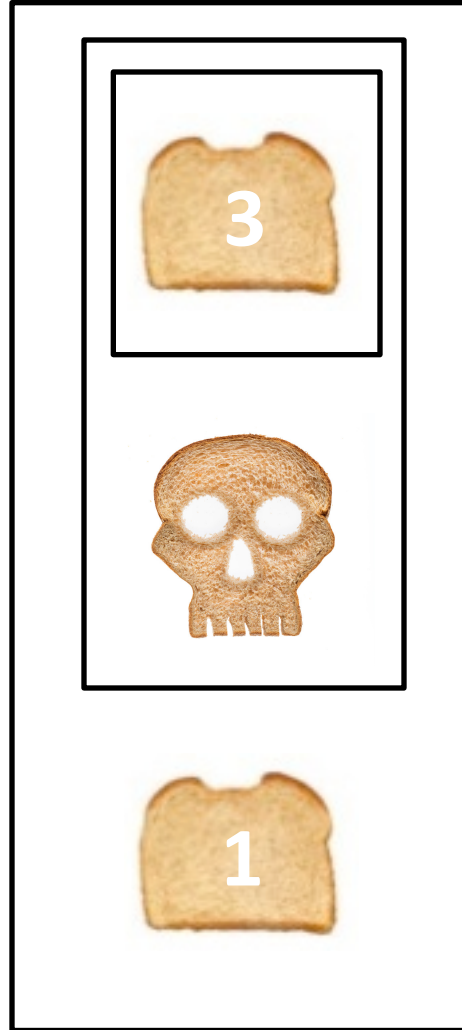


# Hume's Problem, Enhanced.



$$\text{Frntr} \{ \text{bread slice} \} = \{ \text{skull} \}$$

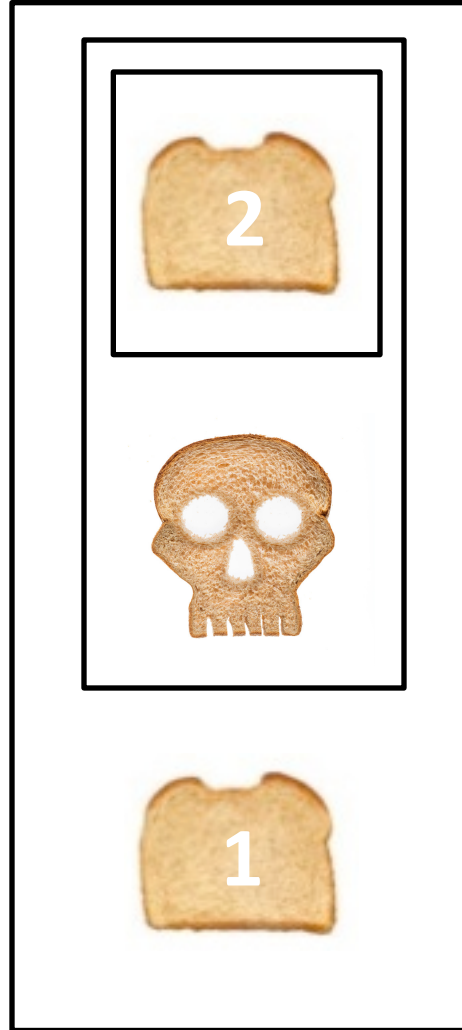
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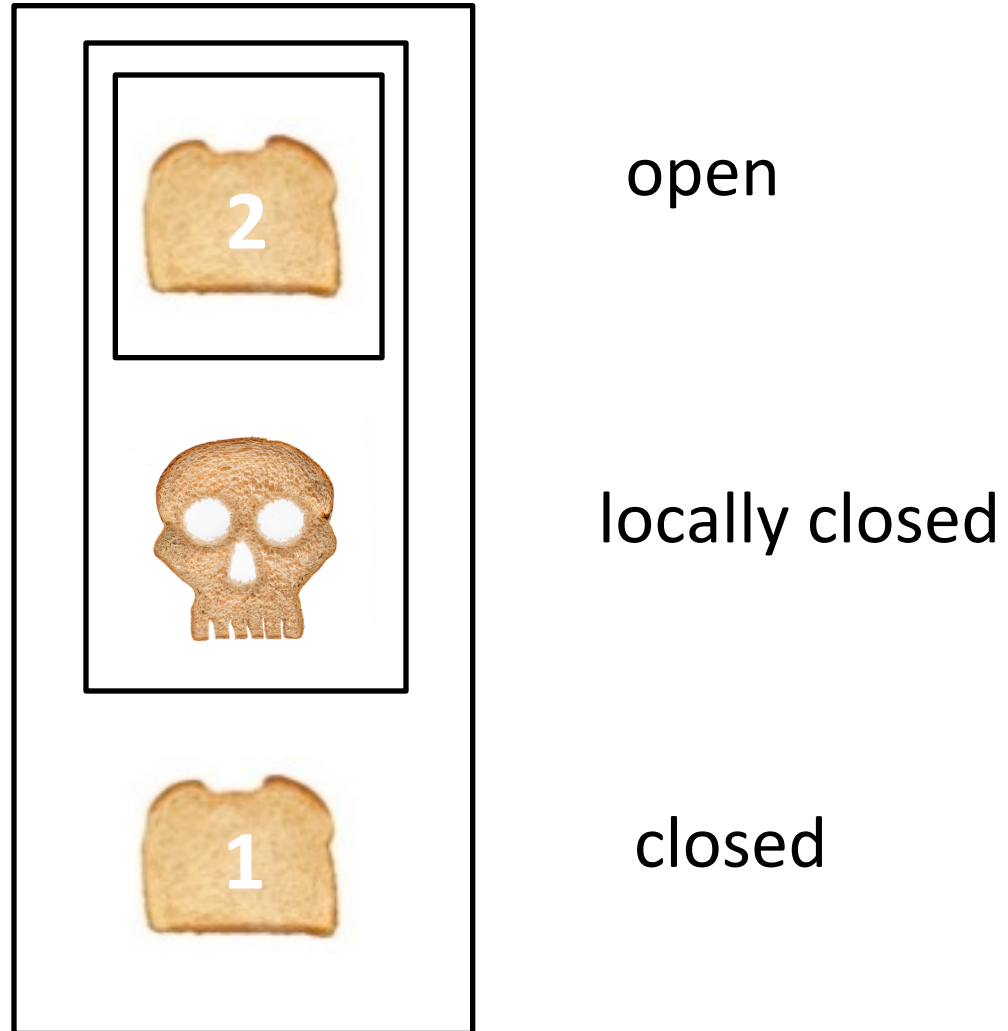
$$\text{Frntr} \{ \text{toast with 2} \} = \{ \text{skull} \}$$

$$\text{Frntr} \{ \text{skull} \} = \{ \text{toast with 1} \}$$

$$\text{Frntr} \{ \text{toast with 1} \} = \emptyset$$

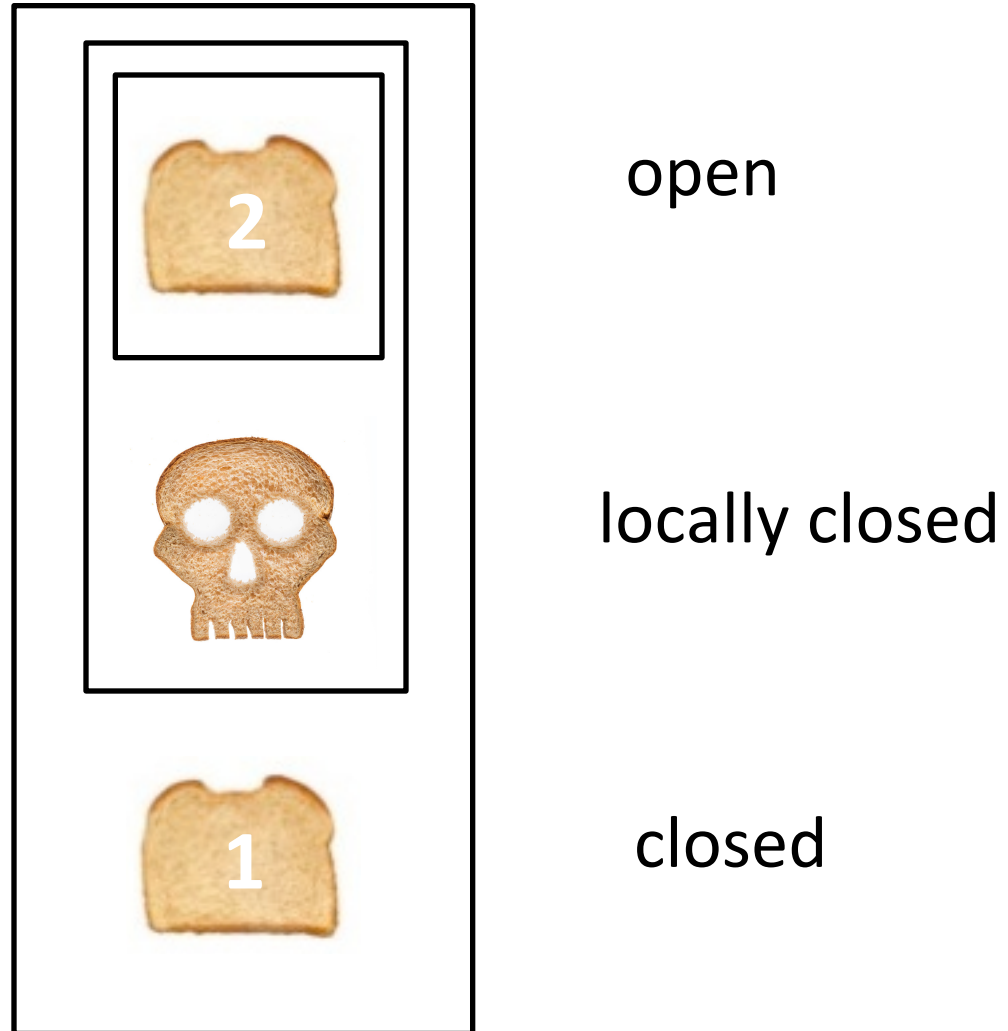
# Locally Closed

$H$  is **locally closed** iff **fr** $\cap$ **tr**  $H$  is closed.



# Locally Closed

$H$  is **locally closed** iff  $H$  entails that  $H$  will be refutable (closed).





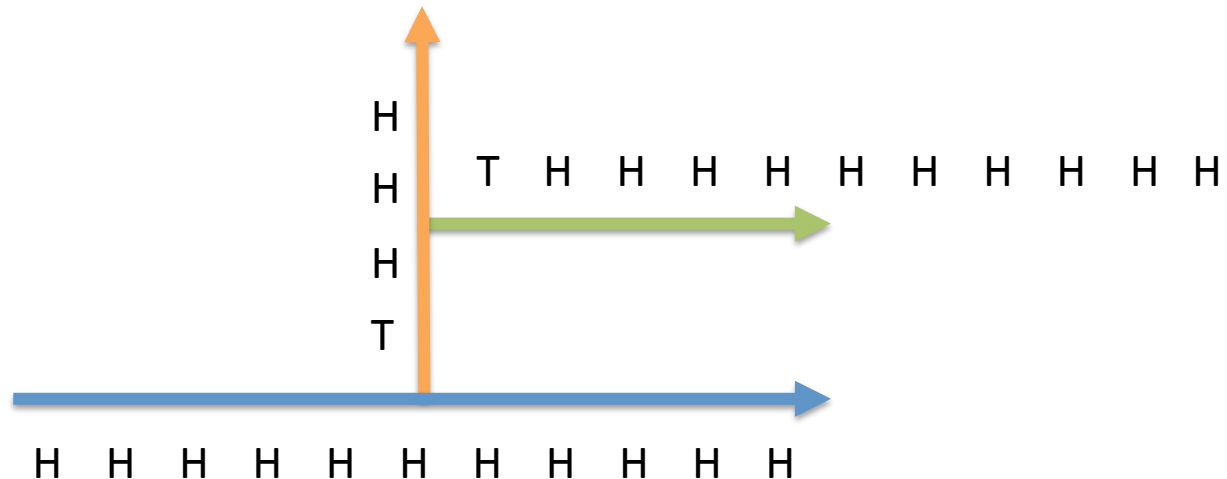
# Sequential Example

*etc.*

$H_2$  = “You will see T exactly twice” is locally closed.

$H_1$  = “You will see T exactly once” is locally closed.

$H_0$  = “You will never see T” is closed.



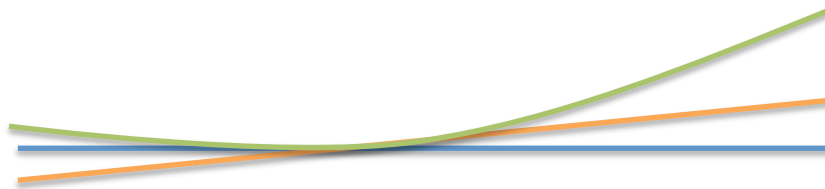
# Equation Example

*etc.*

$H_2$  = “quadratic” is locally closed.

$H_1$  = “linear” is locally closed.

$H_0$  = “constant” is closed.

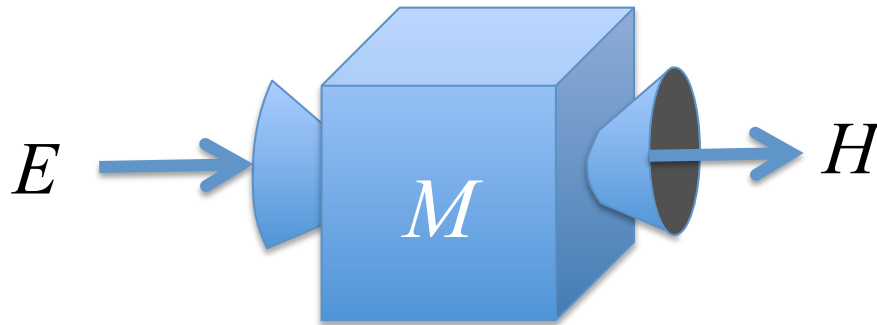


# Topology

- $H$  is **limiting open** iff  $H$  is a countable union of locally closed sets.
- $H$  is **limiting closed** iff  $H^c$  is limiting open.
- $H$  is **limiting clopen** iff  $H$  is both limiting open and limiting closed.

# Propositional Methods

- **Propositional methods** produce **propositional conclusions** in response to **propositional information**.



# Propositional Methods

- $M$  is **infallible** iff  $w \in M(E)$ , whenever  $E \in \mathcal{I}(w)$ .
- $M$  is **monotonic** iff  $M(F) \subseteq M(E)$ , whenever  $F \subseteq E$ .

# Convergence

$M$  converges to  $H$  in  $w$  iff

there is  $E$  in  $\mathcal{I}(w)$  such that

for all  $F$  in  $\mathcal{I}(w \restriction E)$ ,

$$M(F) \subseteq H.$$

# Deductive Methods

- A **verification method** for  $H$  is an infallible, monotonic method  $V$  such that:
  1.  $w \in H^c$  implies  $V(E) = W$  for  $E$  in  $\mathcal{I}(w)$ ;
  2.  $w \in H$  implies  $V$  **converges** to  $H$  in  $w$ .



# Deductive Methods

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  2.  $w \in H$  implies  $V$  **converges** to  $H$  in  $w$ .
- A **refutation method** for  $H$  is just a verification method for  $H^c$ .
- A **decision method** for  $H$  converges to  $H$  or to  $H^c$  without error.

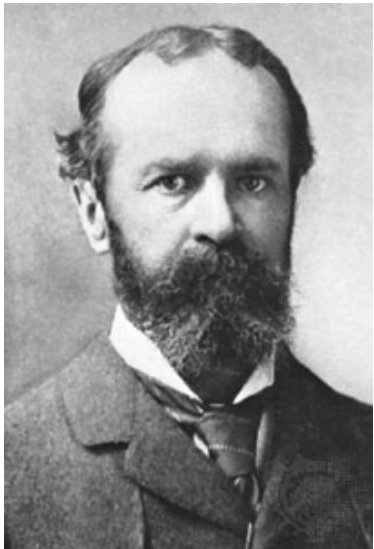


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- A **refutation method** for  $H$  is just a verification method for  $H^c$ .
- A **decision method** for  $H$  converges to  $H$  or to  $H^c$  without error.
- $H$  is **methodologically verifiable [refutable, decidable, etc.]** iff  $H$  has a method of the corresponding kind.

# Inductive Methods

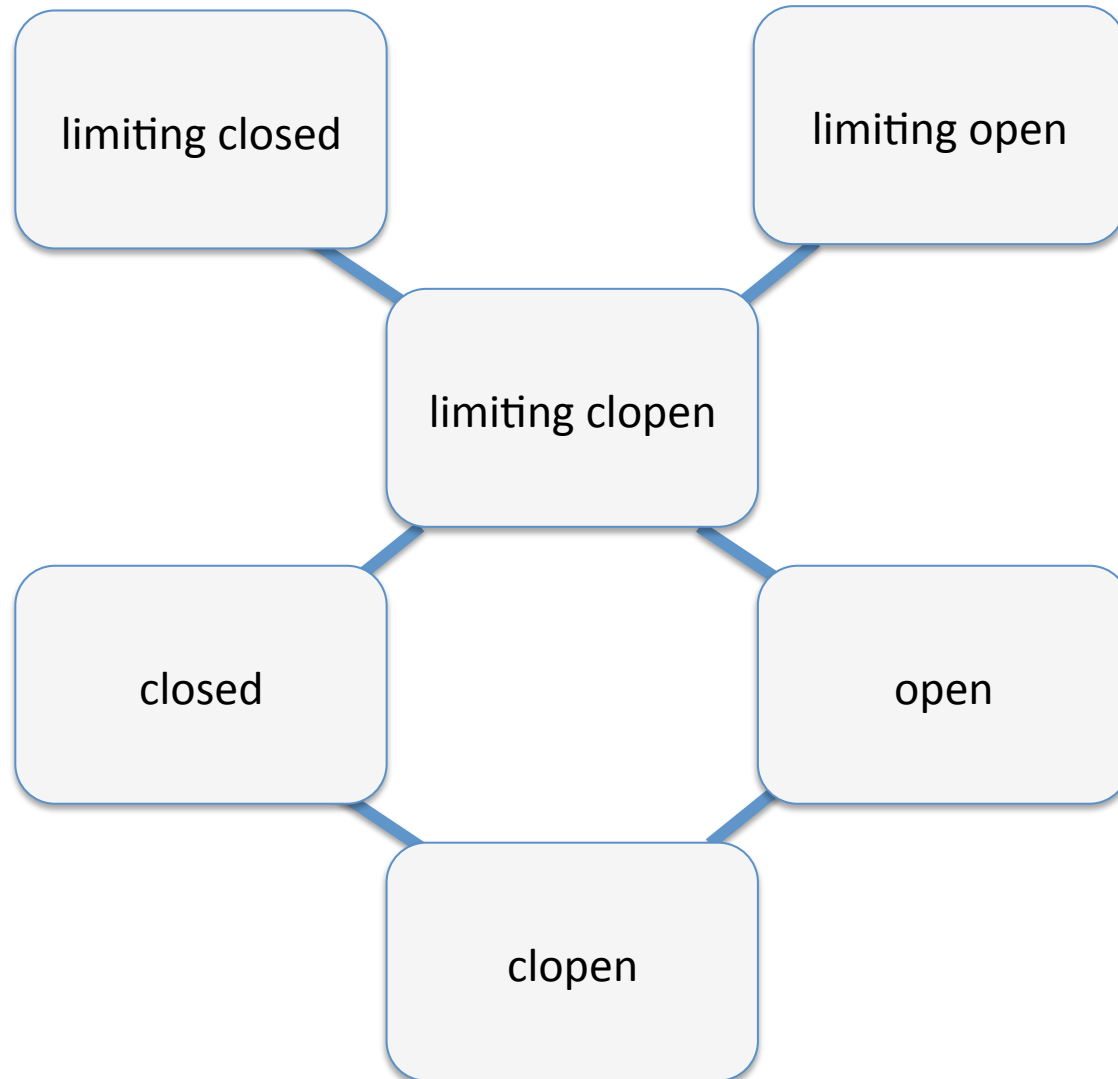
- A **limiting verification method** for  $H$  is a method  $V$  such that:  
 $w \in H$  iff  $V$  converges in  $w$  to some true  $H'$  that entails  $H$ .



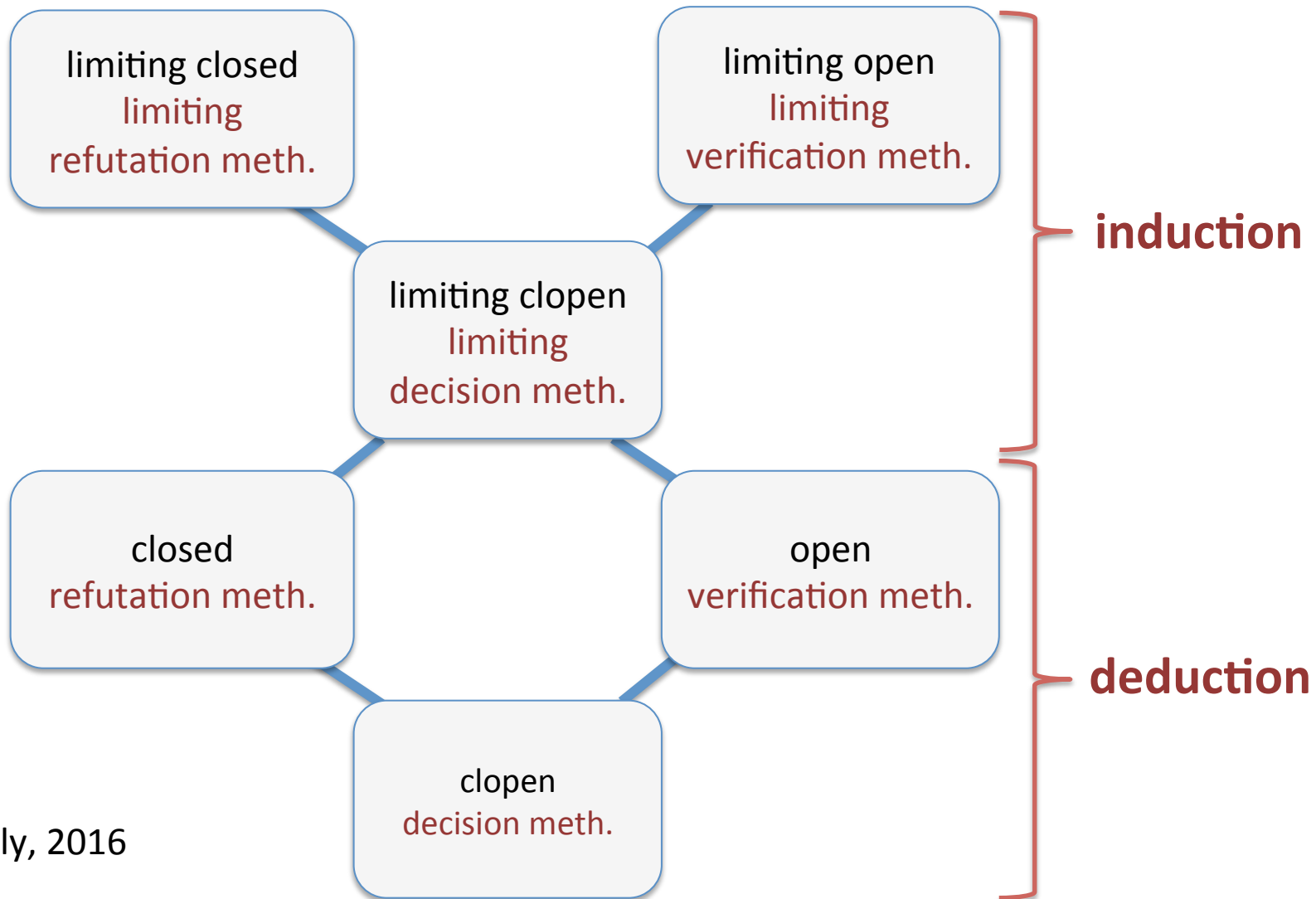
# Inductive Methods

- A **limiting verification method** for  $H$  is a method  $V$  such that:  
 $w \in H$  iff  $V$  converges in  $w$  to some true  $H'$  that entails  $H$ .
- A **limiting refutation method** for  $H$  is a limiting verification method for  $H^c$ .
- A **limiting decision method** for  $H$  is a limiting verification method and a limiting refutation for  $H$ .

# Topological Complexity



# Characterization Theorem



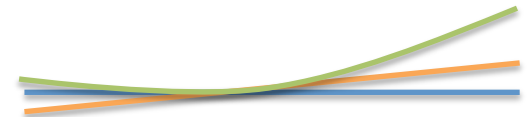
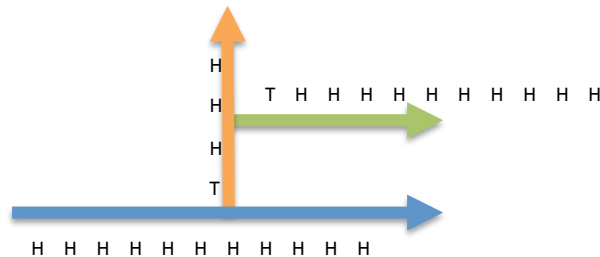


**OCKHAM'S TOPOLOGICAL RAZOR**

# Popper's Simplicity Order

- “More falsifiable hypotheses are simpler”.

$$A \preceq B \Leftrightarrow A \subseteq \text{cl}B.$$



$$H_1 \prec H_2 \prec H_3.$$

# A Big Mistake

$$A \preceq B \Leftrightarrow A \subseteq \text{cl}B.$$

1. Weaker hypotheses are less falsifiable.

$$A \subseteq B \text{ implies } A \preceq B.$$

2. So suspending judgment violates Ockham's razor!

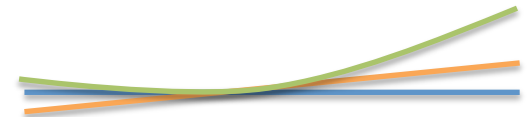
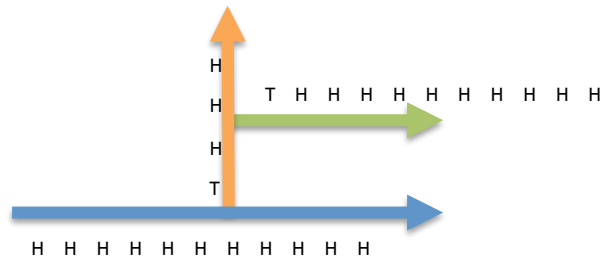
$$A \preceq W.$$



# Easy and Natural Fix

Lack of falsifiers is **bad** only if  $A$  is **false**!

$$A \preceq B \Leftrightarrow A \subseteq \text{frntr} B$$



$$H_1 \prec H_2 \prec H_3.$$

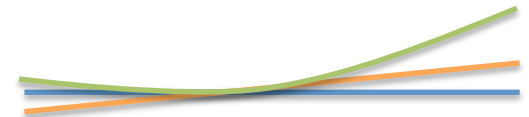
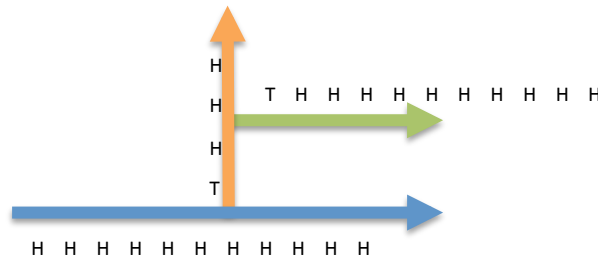
# A Smaller Issue

- Gerrymandered hypotheses can obscure simplicity relations.
- E.g., “The true law is linear, or the cat is on the mat” is not simpler than “The true law is quadratic”.

# A Response

Simpler theories have simpler ways of being true.

$$A \triangleleft B \Leftrightarrow A \cap \text{frntr} B \neq \emptyset$$



$$H_1 \triangleleft H_2 \triangleleft H_3.$$

# Example: Competing Paradigms

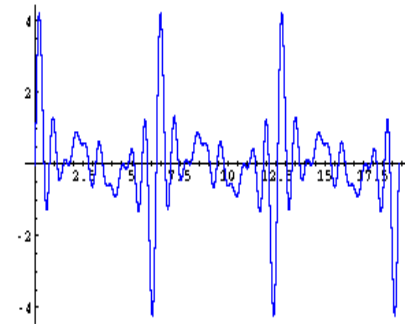
Polynomial paradigm

$$Y = \sum_{i=0}^N a_i X^i.$$



Trigonometric polynomial paradigm

$$Y = \sum_{i=0}^N a_i \sin(iX) + b_i \cos(iX).$$



# Example: Competing Paradigms

Polynomial paradigm

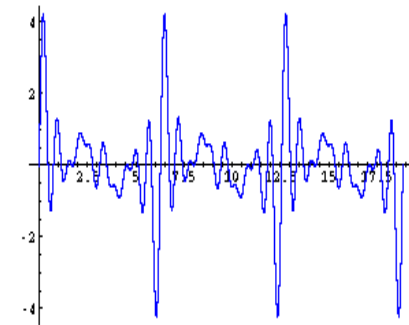
$$Y = \sum_{i=0}^N a_i X^i.$$

degree



Trigonometric polynomial paradigm

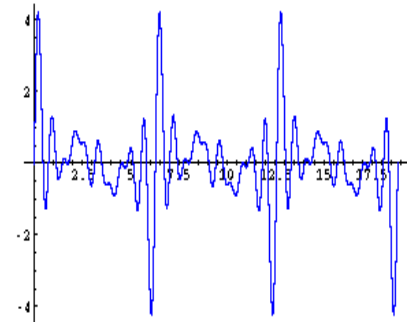
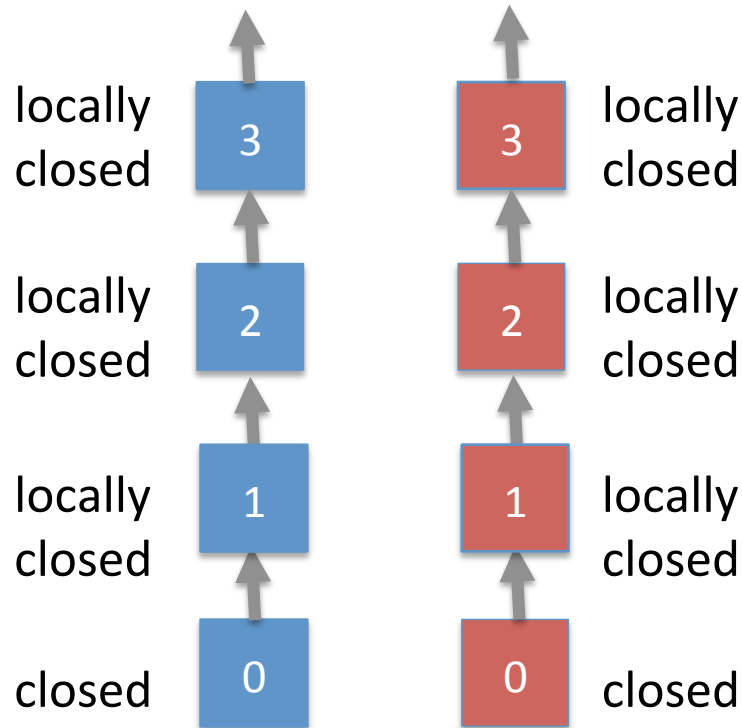
$$Y = \sum_{i=0}^N a_i \sin(iX) + b_i \cos(iX).$$



# Example: Competing Paradigms

$\mathcal{Q}$  = which degree and which paradigm is true?

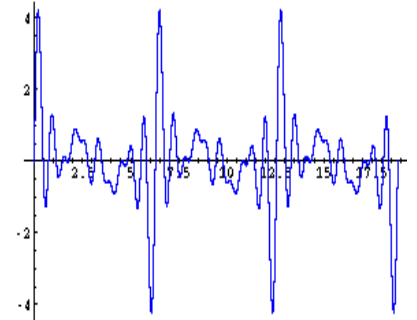
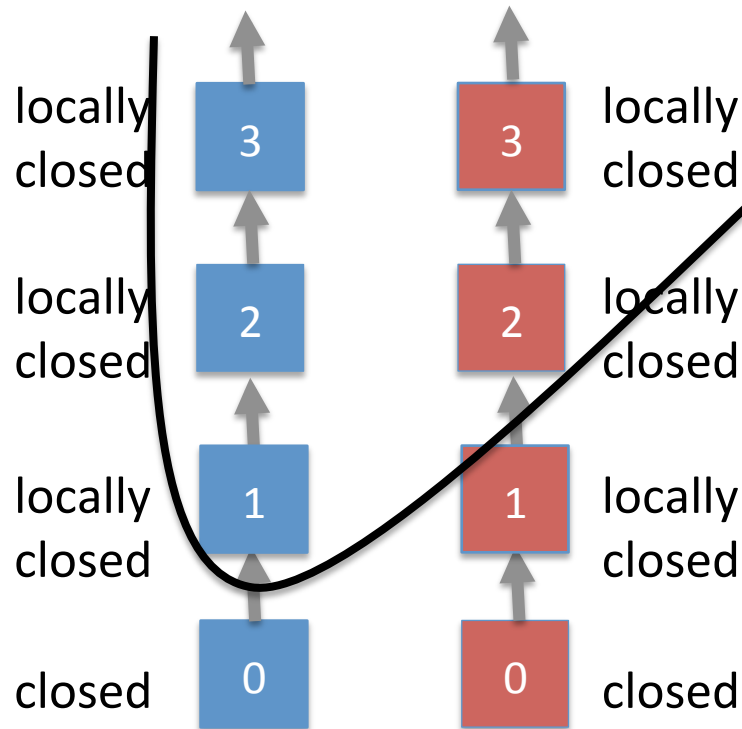
$\mathcal{I}$  = finitely many inexact measurements.



# Example: Competing Paradigms

$\mathcal{Q}$  = which degree and which paradigm is true?

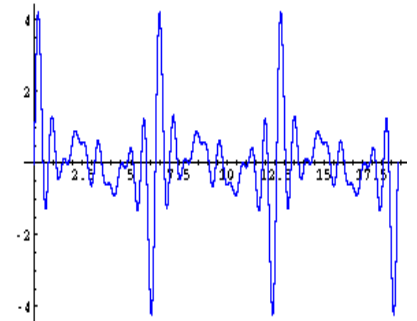
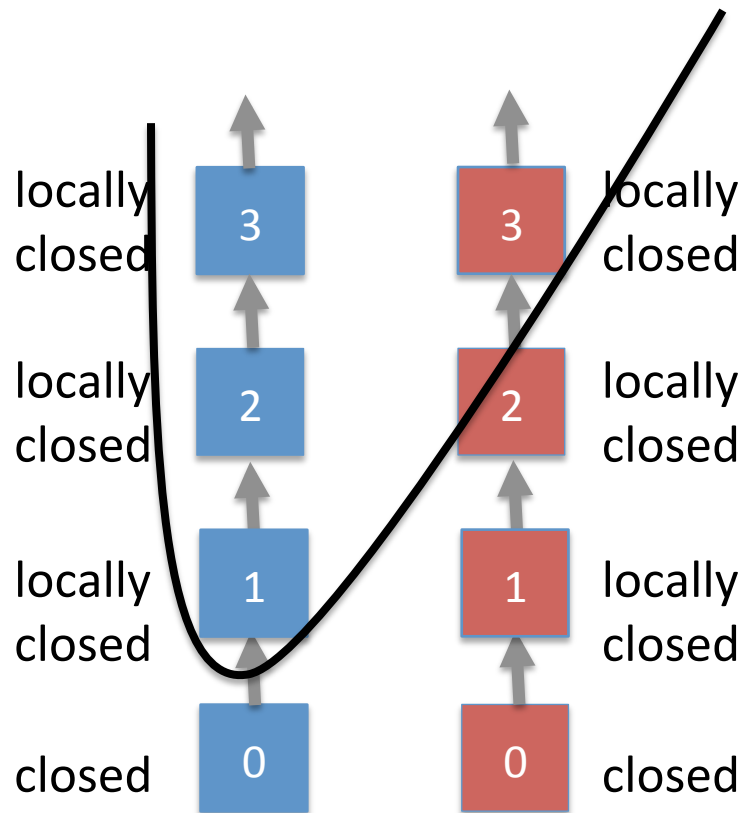
$\mathcal{I}$  = finitely many inexact measurements.



# Example: Competing Paradigms

$\mathcal{Q}$  = which degree and which paradigm is true?

$\mathcal{I}$  = finitely many inexact measurements.

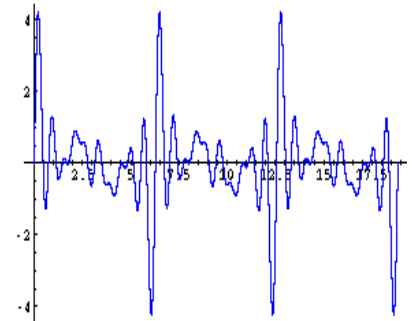
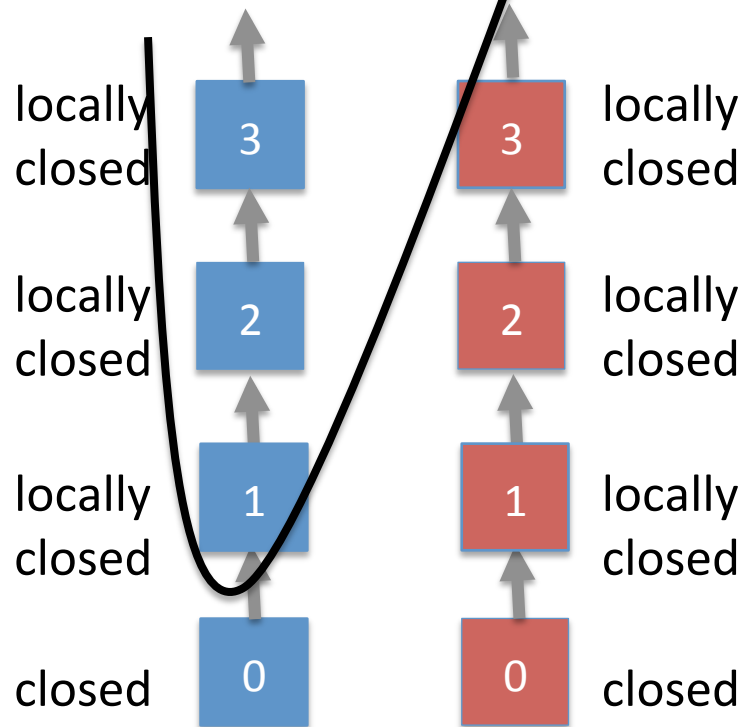




# Example: Competing Paradigms

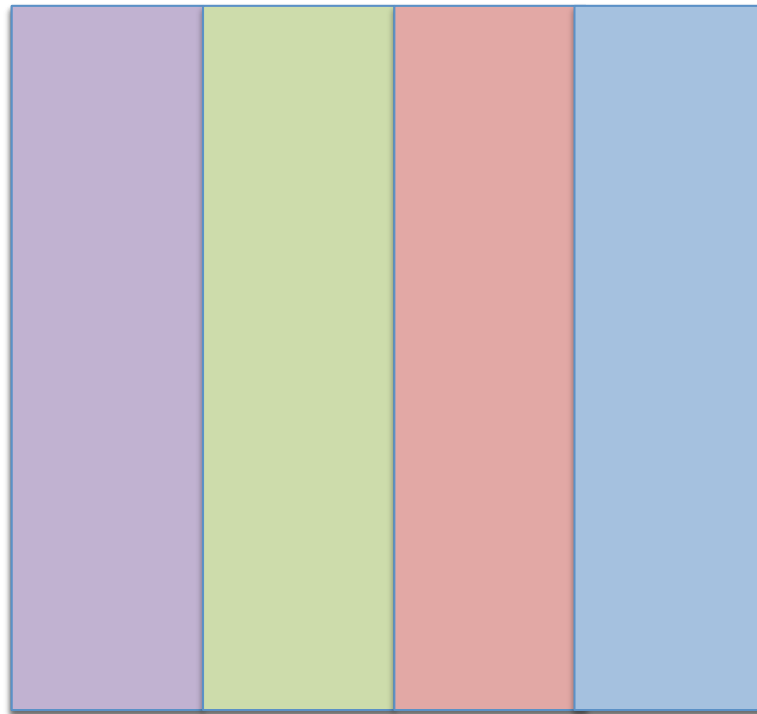
$\mathcal{Q}$  = which degree and which paradigm is true?

$\mathcal{I}$  = finitely many inexact measurements.



# Questions

- A **question** partitions  $W$  into countably many possible answers (Hamblin 1958)
- **Relevant responses** are disjunctions of answers.



# Ockham's Razor

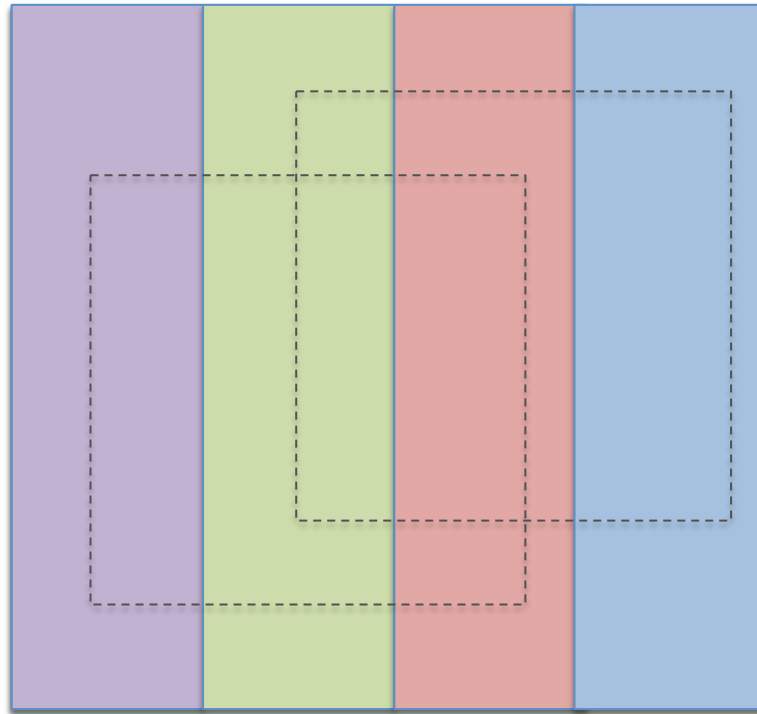


**Proposition (Genin and Kelly, 2016).** The following principles are **equivalent**.

1. Infer a **simplest** relevant response in light of  $E$ .
2. Infer a **refutable** relevant response compatible with  $E$ .
3. Infer a relevant response that is **not more complex than the true answer**.

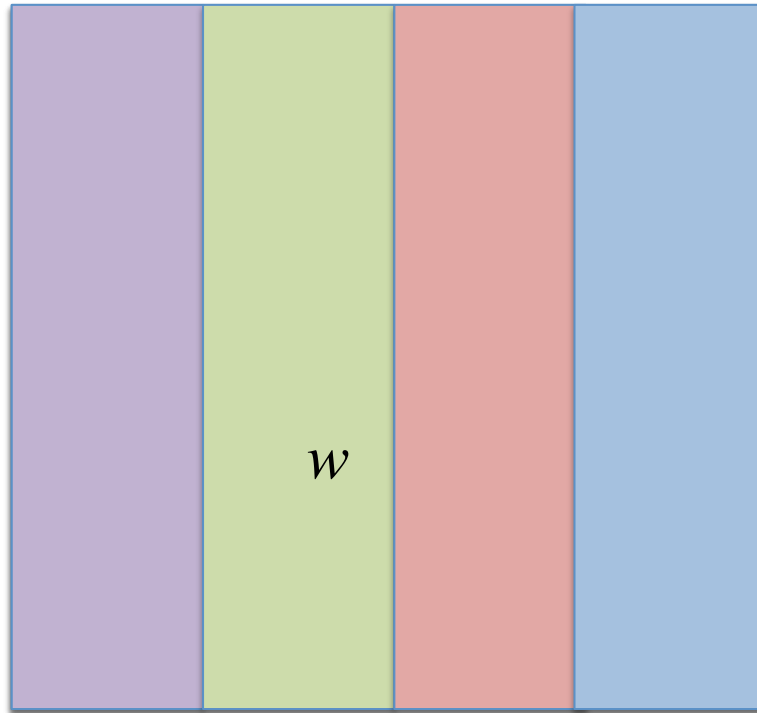
# Empirical Problem

$$\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q}).$$



# Empirical Problem

$Q(w)$  is the answer true in  $w$ .



# Solutions

A **solution** for  $\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$  is a propositional method  $V$  such that

$w \in H$  iff  $V$  converges in  $w$  to some true  $H'$  that entails  $\mathcal{Q}(w)$ .

A problem is **solvable** iff it has a solution.

# Solvability, Characterized.

**Proposition.** A problem  $\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$  is solvable iff every answer is limiting open.

de Brecht and Yamamoto (2009)

Baltag, Gierasimczuk, and Smets (2015)

Genin and Kelly (2015)

# Progressive Solutions

A solution for  $\mathfrak{P} = (W, \mathcal{I}, Q)$  is **progressive** iff for all  $E$  in  $\mathcal{I}(w)$  and  $F$  in  $\mathcal{I}(w \mid E)$  :

if  $V(E)$  entails  $Q(w)$ , then  $V(F)$  entails  $Q(w)$ .

That is: the true answer is a **fixed point** of inquiry.



# Progressive Solutions

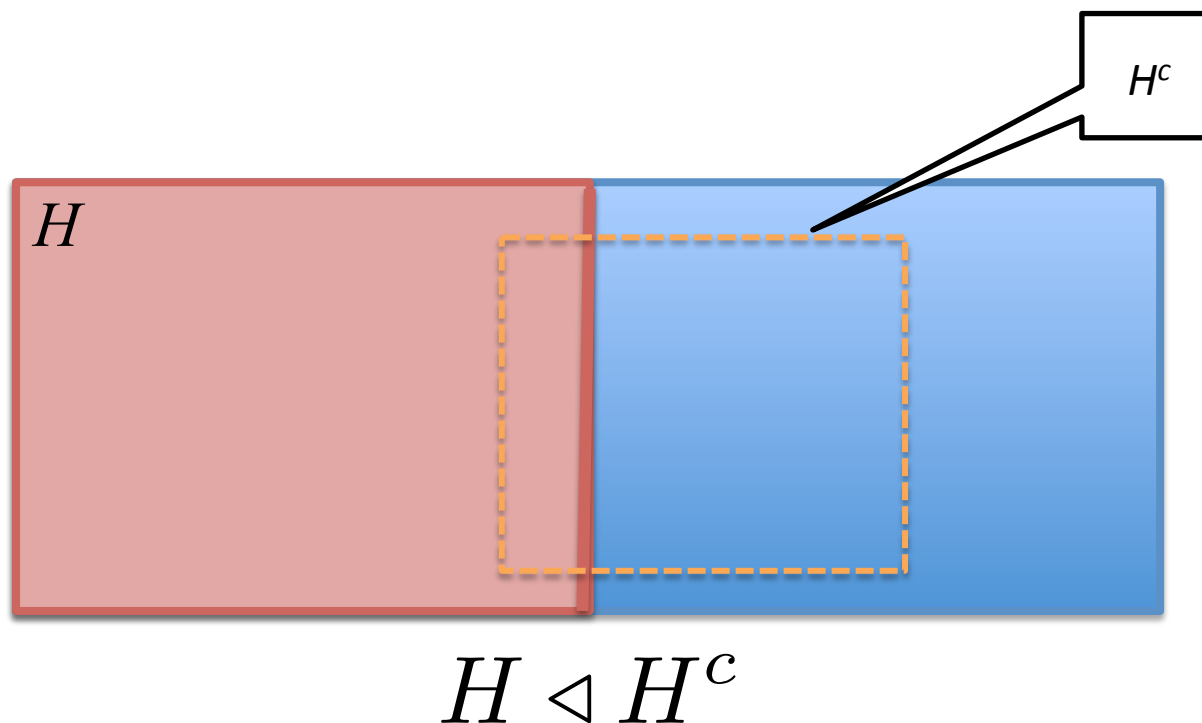
**Proposition.** If there exists an enumeration  $A_1, A_2, \dots$  of the answers to  $Q$  agreeing with the **simplicity order**, then  $Q$  is progressively solvable.

# Epistemic Mandate for Ockham's Razor

**Proposition** (Genin and Kelly, 2016). Every progressive solution obeys Ockham's razor.

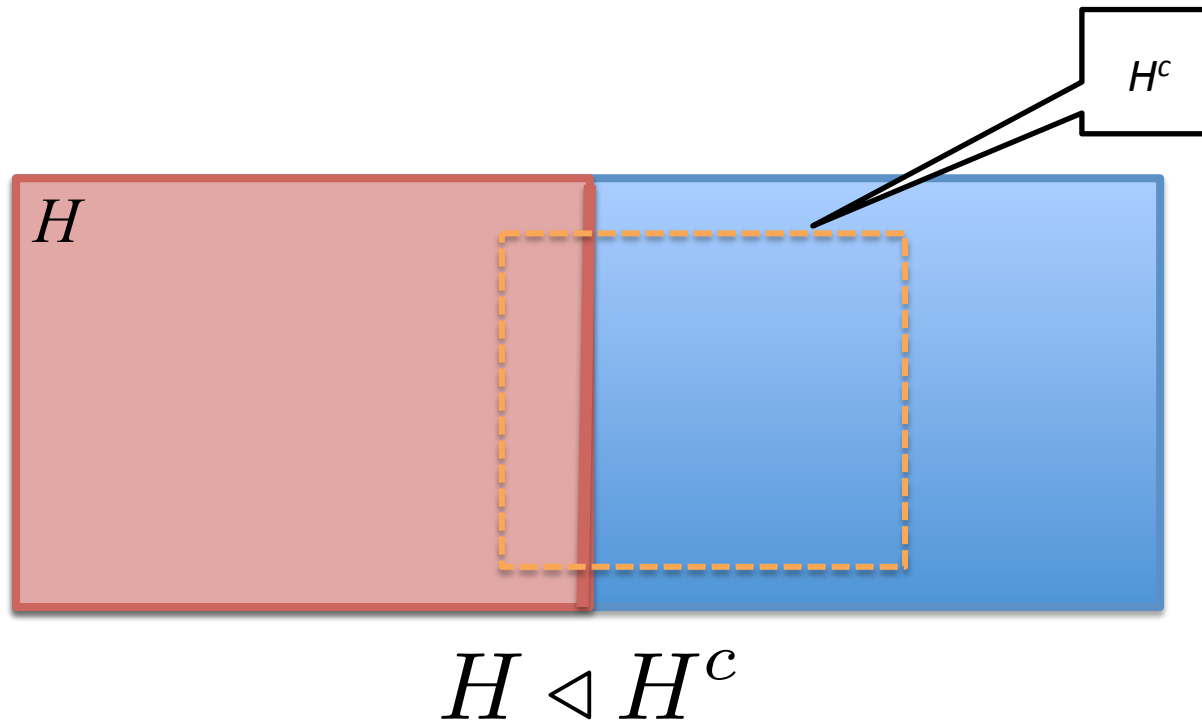
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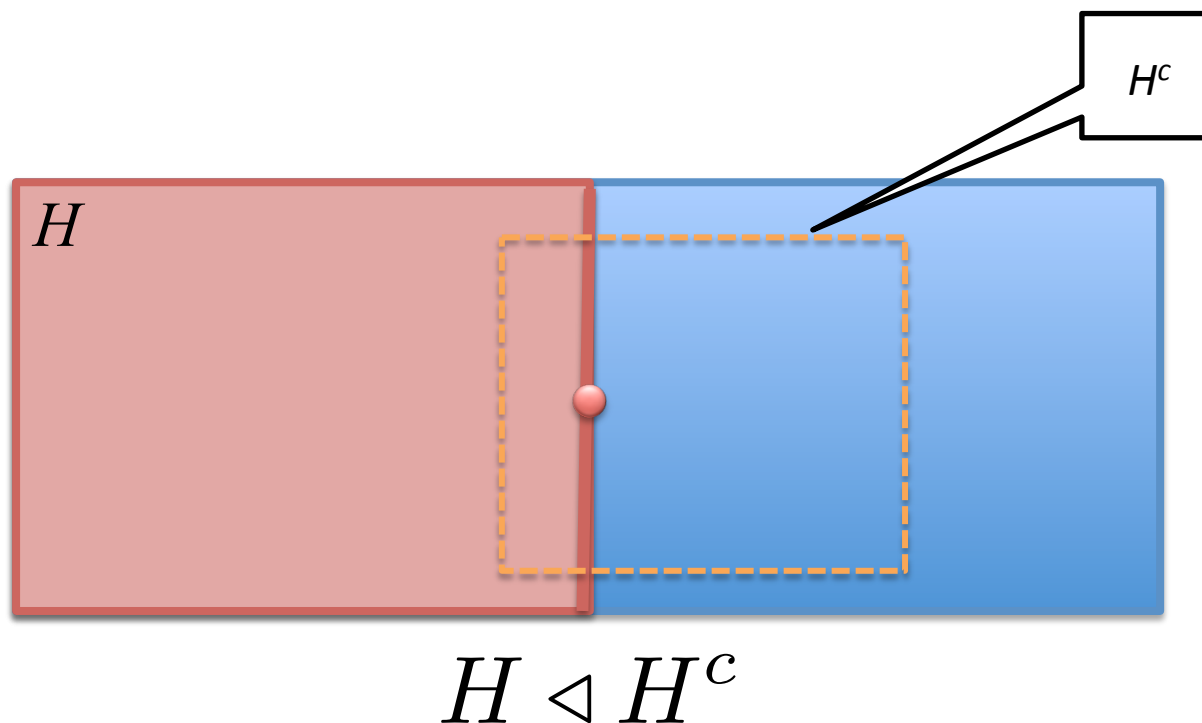
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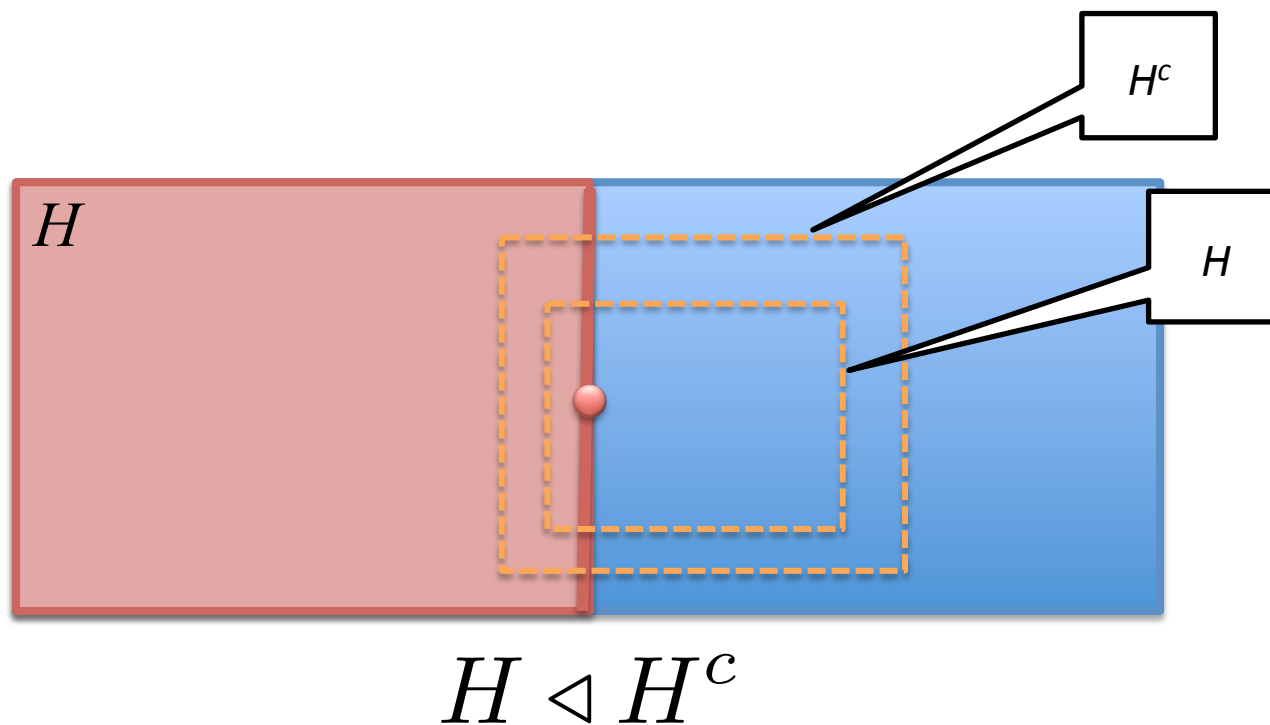
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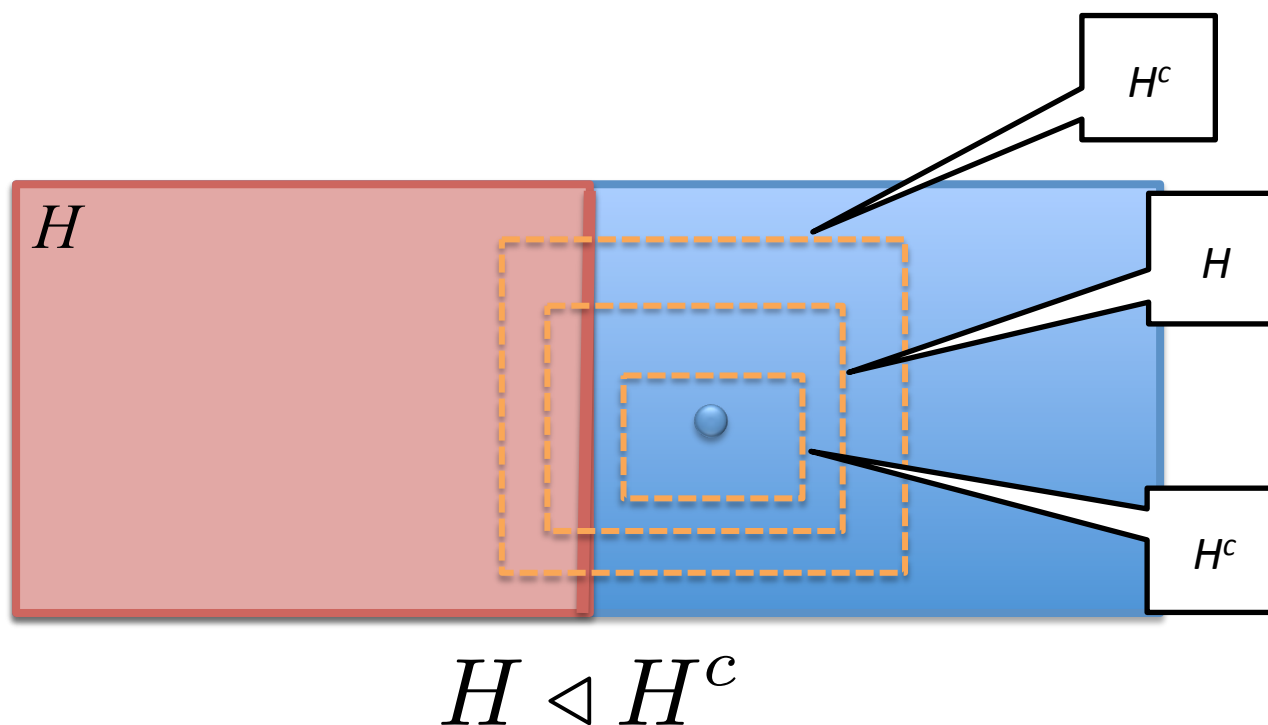
# Epistemic Mandate for Ockham's Razor

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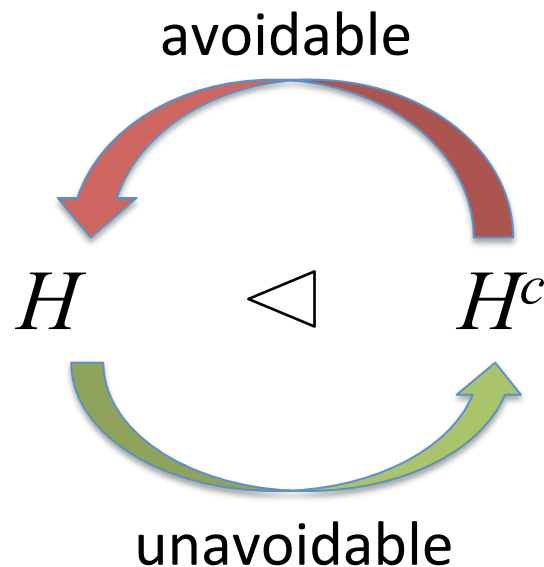
# Epistemic Mandate for Ockham's Razor

**Proposition** (Genin and Kelly, 2016). Every progressive solution obeys Ockham's razor.



# Non-Circular

By **favoring** a **complex** hypothesis, you lose in a **complex** world!





# Skepticism

That story

“... may be okay if the candidate theories are **deductively** related to observations, but when the relationship is **probabilistic**, I am skeptical ...”

Elliot Sober (2015).



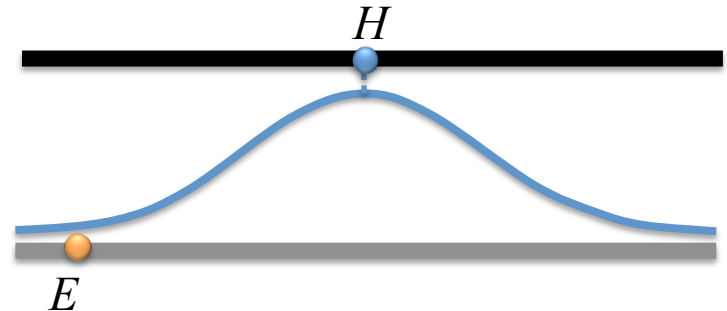
# A Worry

- Propositional information refutes logically incompatible possibilities.



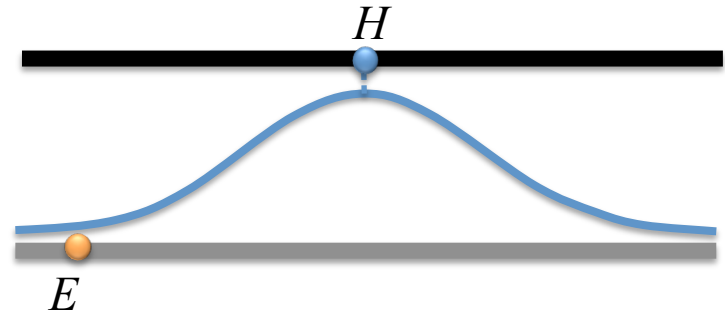
# A Worry

- Propositional information refutes logically incompatible possibilities.
- Typically, statistical samples are logically compatible with **every** possibility.



# Response

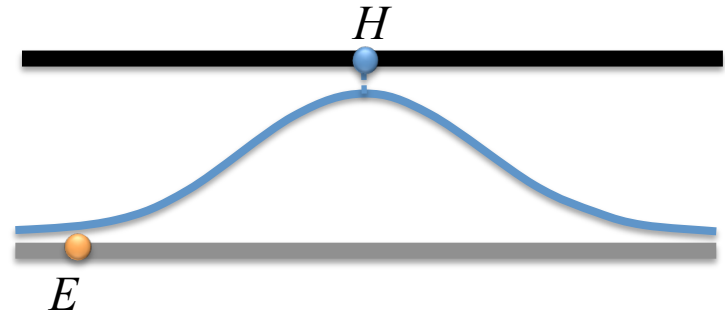
Don't worry!



# Response

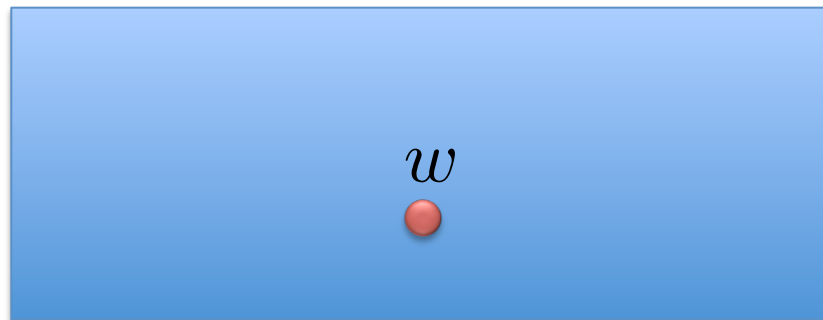
Don't worry!

Common **topological** structure



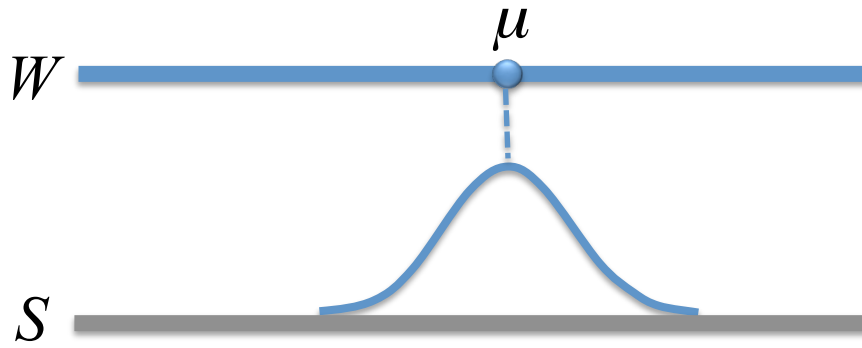
# Recall: Possible Worlds

$W$



# Statistical Worlds

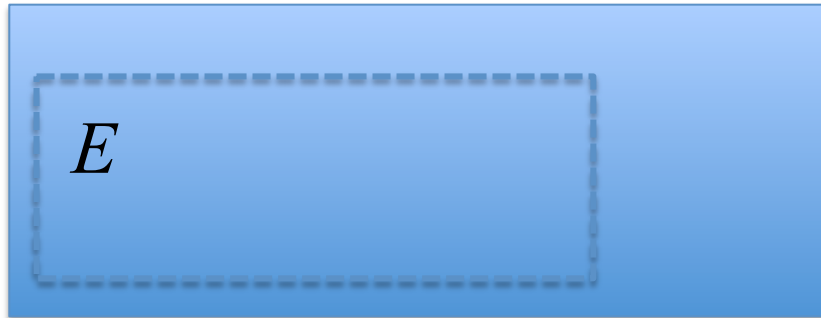
- Probability measures over a sample space.



# Recall: Information States

The **logically strongest** proposition you are informed of.

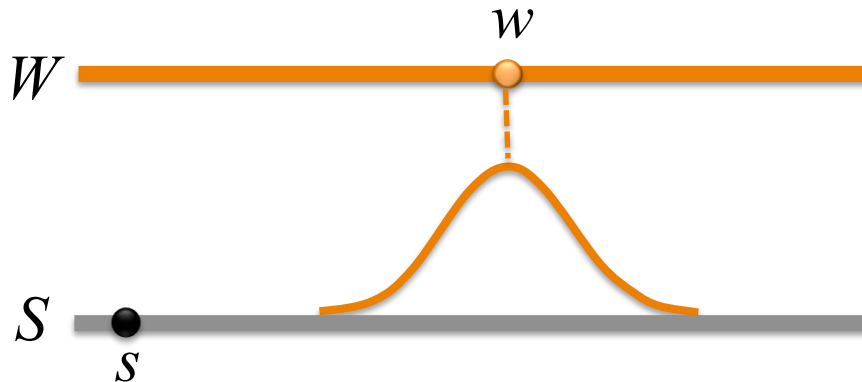
$W$



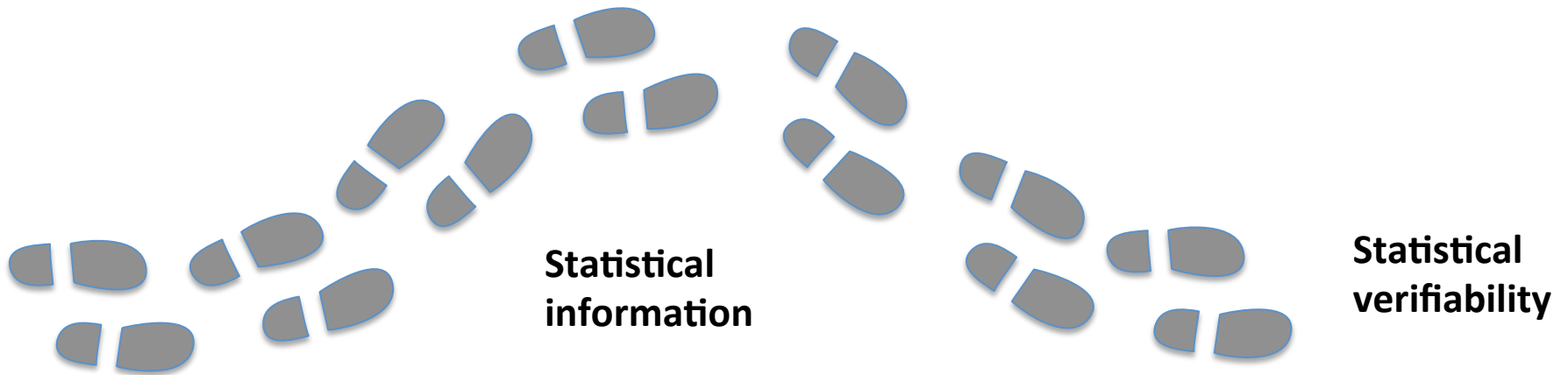


# Statistical Information?

- It seems that the only statistical information state is  $W$ .

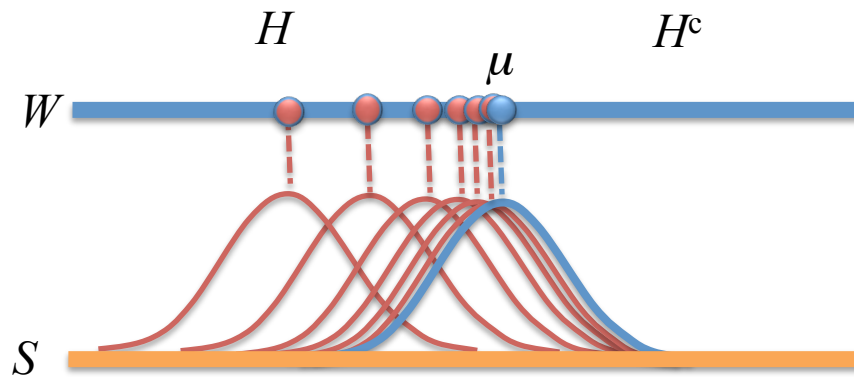


# Side-step the Worry



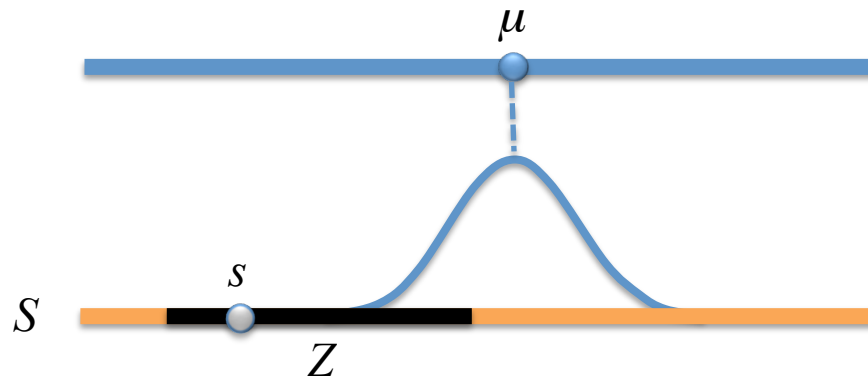
# Statistical Information Topology

Possibilities **nearer** to the truth should be **harder** to **rule out** by **statistical** methods.



# Gathering Statistical Information

1. The **sample space**  $S$  has its own topology.
2. Choose a **sample event**  $Z$  over  $S$ .
3. Obtain sample  $s$ .
4. Observe whether  $Z$  **occurs**.

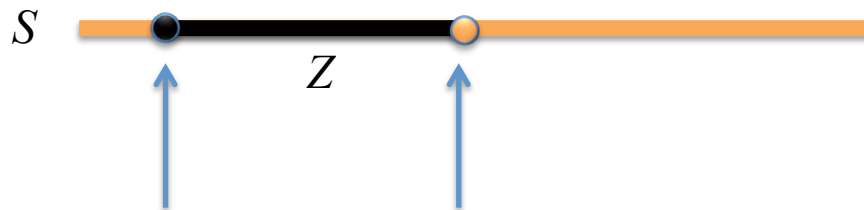
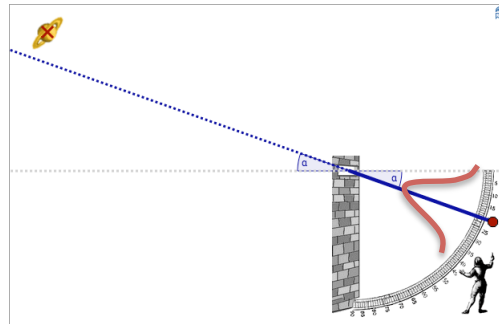


# Feasible Sample Events

- You can't **decide** whether a sample is rational-valued.

# Feasible Sample Events

- You can't **determine** whether a sample hits **exactly** on the **boundary** of an open interval.



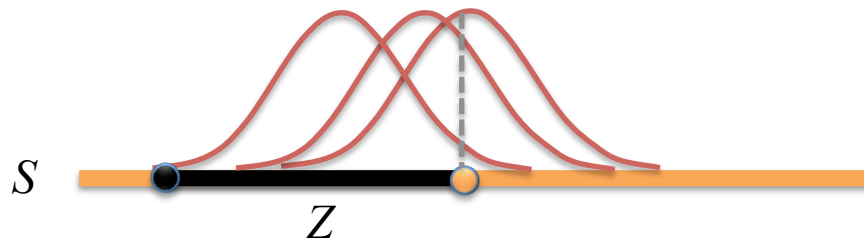
# Feasible Sample Events

- But **every** non-trivial  $Z$  on the real line has **boundary points**.



# Feasible Sample Events

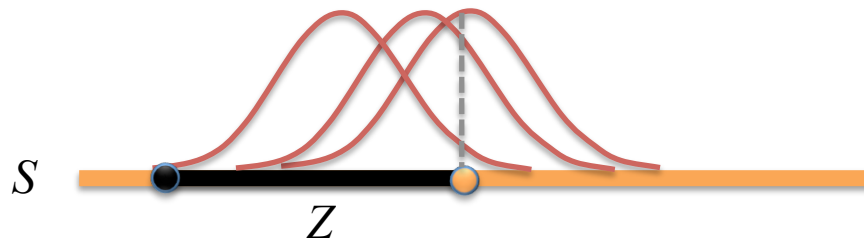
- That doesn't matter **statistically** as long as the boundary carries 0 probability.
- So  $Z$  is a **feasible** sample event iff
$$p(\text{bdry } Z) = 0, \text{ for each } p \text{ in } \mathcal{W}.$$
- I.e, **feasible**  $Z$  is **almost surely clopen** (decidable) in  $S$ .





# Feasible Statistical Models

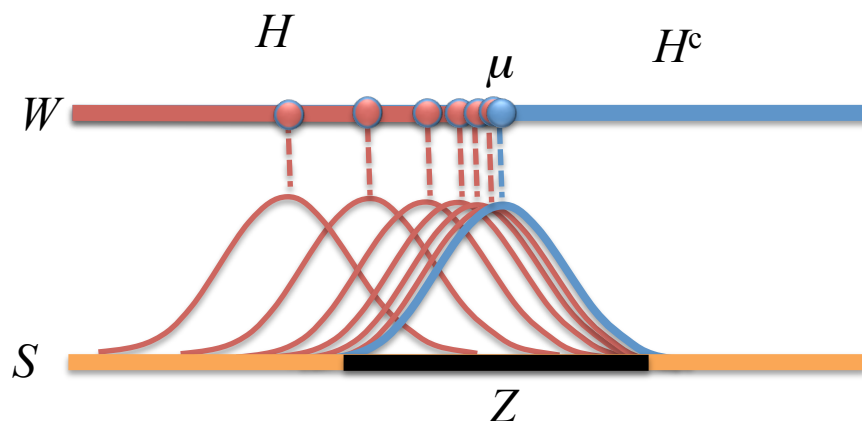
- $S$  is **feasible** for  $W$  iff  
 $S$  has a **countable topological basis of feasible zones**.



# Statistical Information Topology

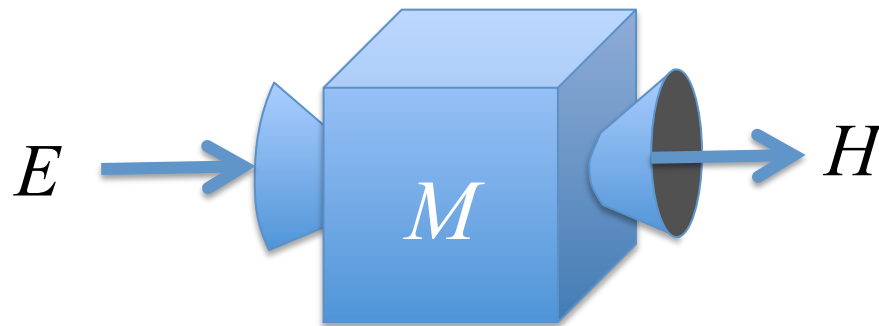
$w \in \text{cl}(H)$  iff  $H$  contains a sequence of worlds  $\mu_1, \dots, \mu_n, \dots$  such that for **every feasible** sample event  $Z \subseteq S$ :

$$\lim_{n \rightarrow \infty} \mu_n(Z) \rightarrow \mu(Z).$$



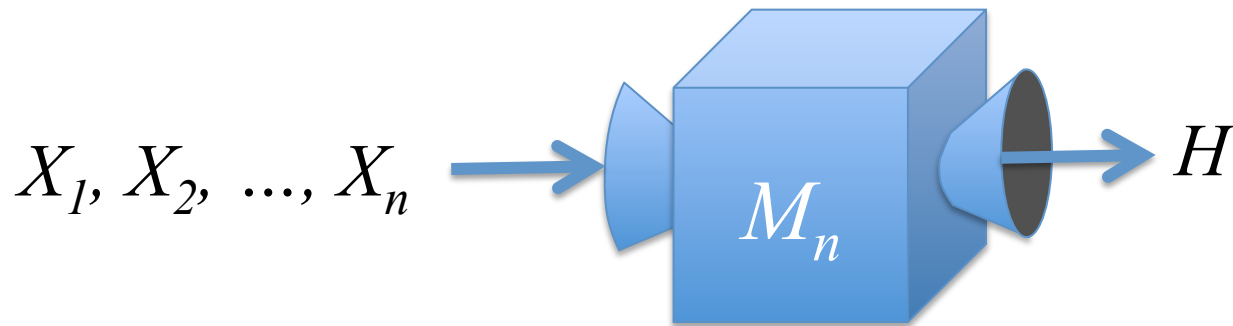
# Recall: Propositional Methods

- **Propositional methods** produce **propositional conclusions** in response to **propositional information**.



# Statistical Methods

- **Statistical methods** produce **propositional conclusions** in response to **statistical samples**.



# Feasible Statistical Methods

A **feasible statistical method** at sample size  $n$  is a function  $M_n$  from sample events in  $S^n$  to **propositions** over  $W$  such that:

$(M_n)^{-1}(H)$  is **feasible**.

A **feasible statistical method** is a collection

$(M_n : n \in \mathbf{N})$

of feasible statistical methods at each sample size.

# Recall: Verification Methods

- A **verification method** for  $H$  is an infallible, monotonic method  $V$  such that:
  1.  $w \in H^c$  implies  $V$  always concludes  $W$ .
  2.  $w \in H$  implies  $V$  **converges** to  $H$ .



# Statistical Verification

- A **statistical verification method** for  $H$  at **significance level**  $\alpha > 0$  is a feasible method ( $V_n : n \geq 1$ ), such that:
  1. at each sample size, outputs  $W$  with probability at least  $1-\alpha$ , if  $H$  is **false**.
  2. converges in probability to  $H$ , if  $H$  is **true**.
- $H$  is **statistically verifiable** iff  $H$  has a **statistical verification method** at **each**  $\alpha > 0$ .

# Statistical Verification

- A **statistical verification method** for  $H$  at **significance level**  $\alpha > 0$  is a feasible method  $(V_n : n \geq 1)$ , such that:
  1.  $\mu^n [V_n^{-1}(W)] \geq 1 - \alpha$ , if  $H$  is false in  $\mu$ ;
  2.  $\mu^n [V_n^{-1}(H)] \rightarrow 1$ , if  $H$  is true in  $\mu$ .
- $H$  is **statistically verifiable** iff  $H$  has a **statistical verification method** at **each**  $\alpha > 0$ .



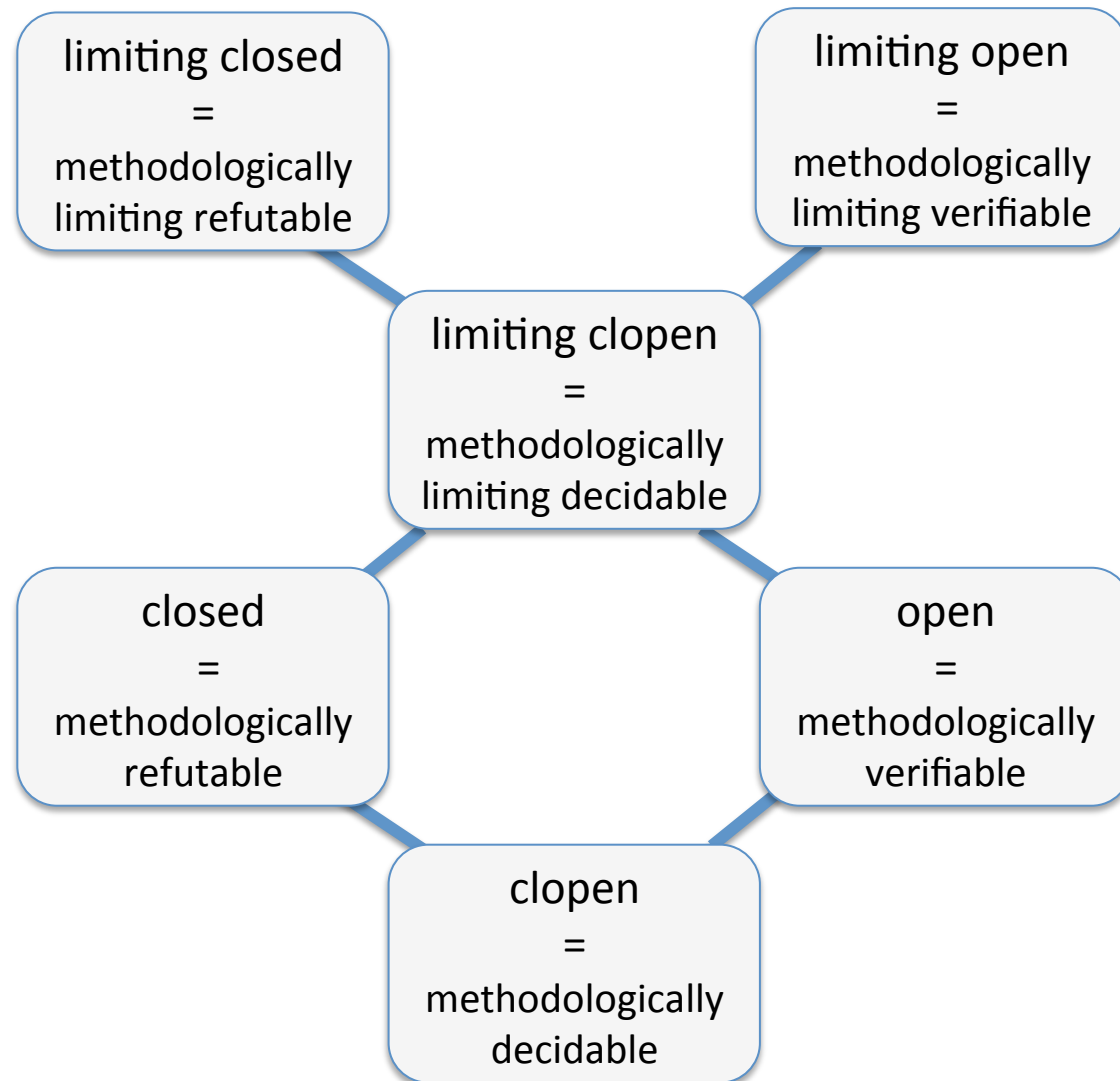
# Recall: Verification in the Limit

- A **limiting verification method** for  $H$  is a method  $M$  such that in **every** world  $w$ :  
 $H$  is true in  $w$  iff  $M$  converges to some true  $H'$  that entails  $H$ .
- $H$  is **verifiable in the limit** iff  $H$  has a limiting verifier.

# Statistical Verification in the Limit

- A **limiting statistical verification method** for  $H$ 
  - converges in probability to some  $H'$  entailing  $H$  iff  $H$  is true.
- $H$  is **statistically verifiable in the limit** iff  $H$  has a limiting **statistical** verifier.

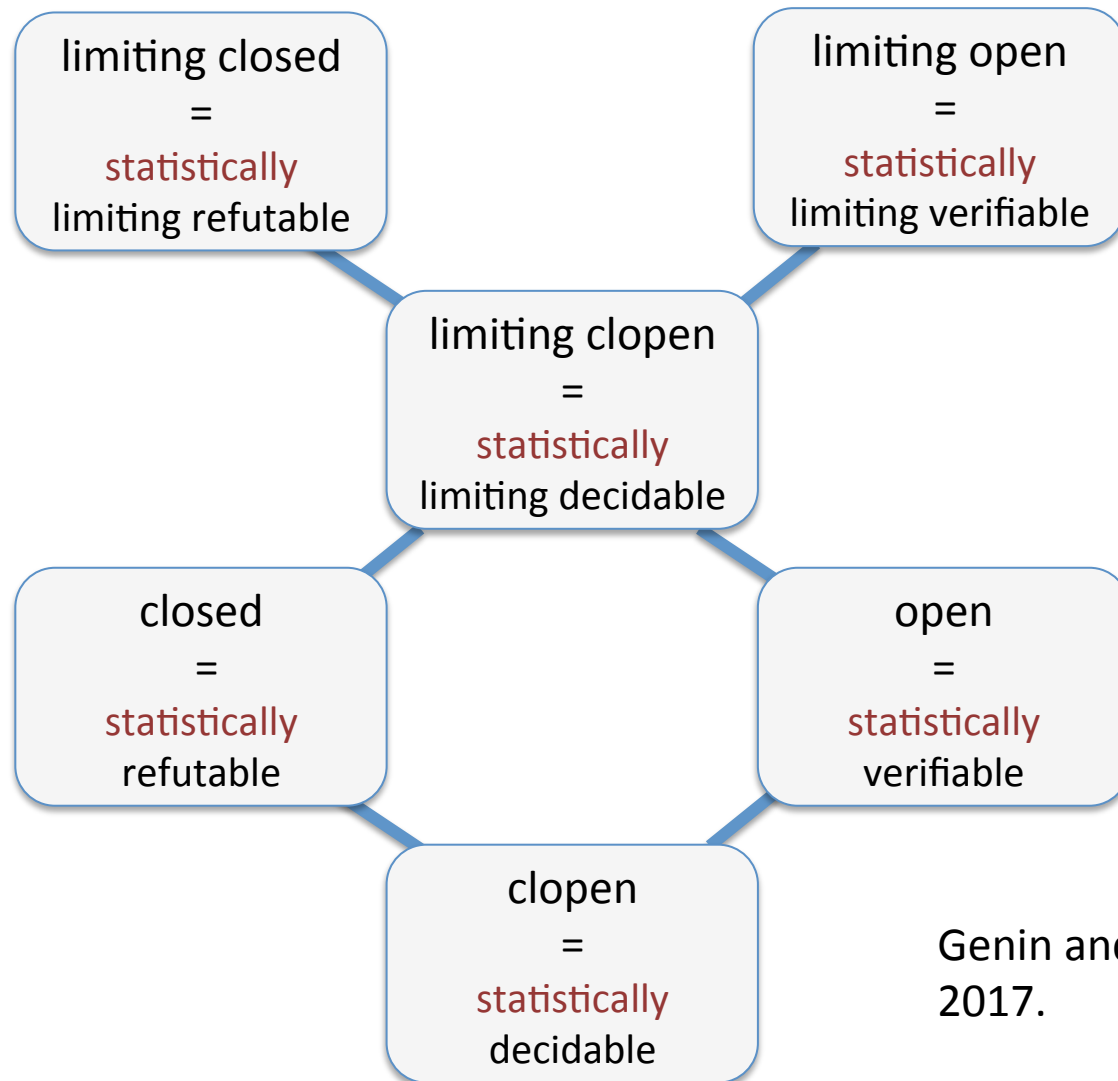
# The Propositional Hierarchy



# The Main Result

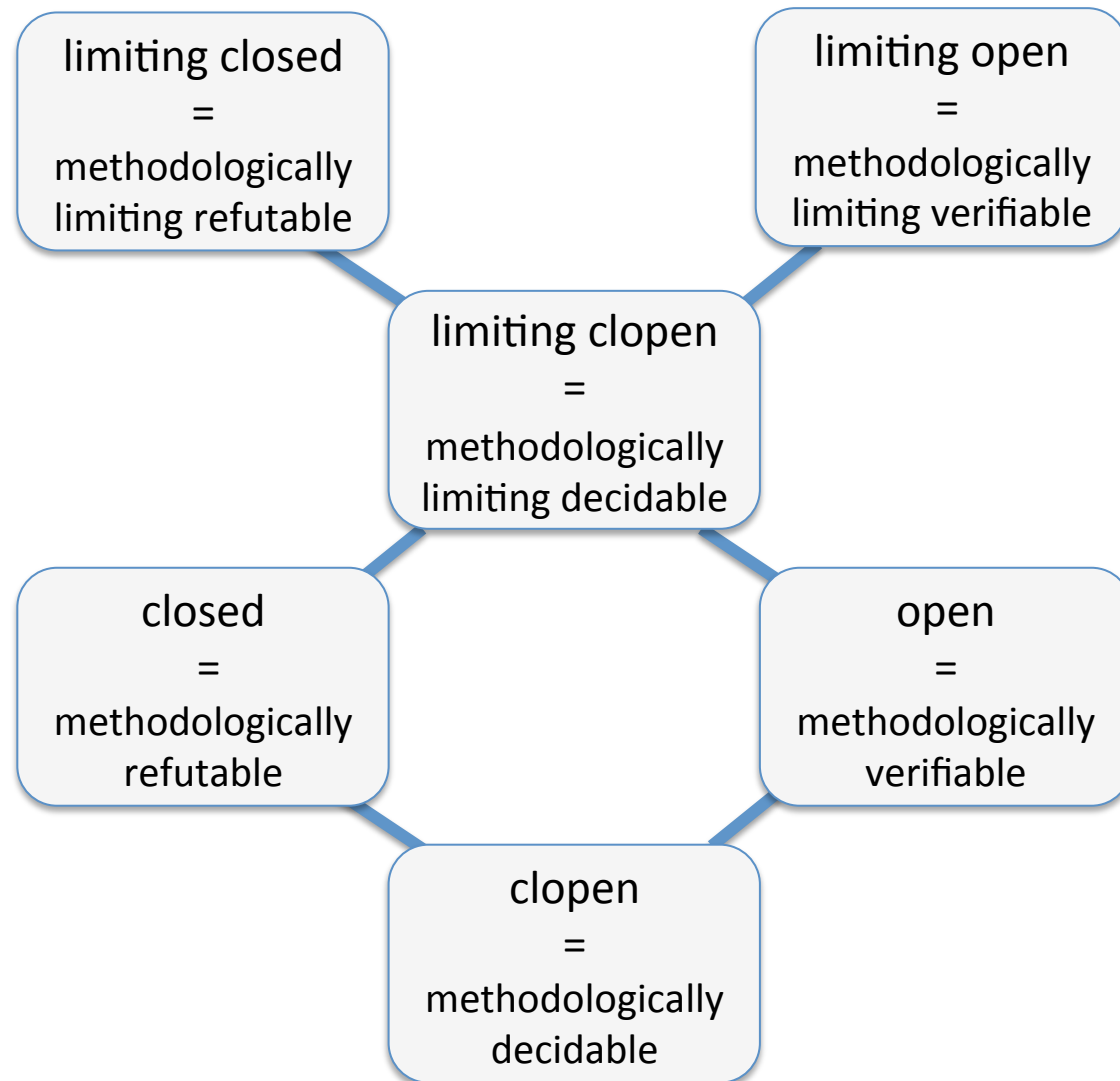
**Proposition.** (Genin, Kelly 2017) Suppose that  $S$  is feasible for  $W$ . Then, the open sets in the weak topology are exactly the statistically verifiable hypotheses.

# The Statistical Hierarchy



Genin and Kelly,  
2017.

# So in **Both** Logic and Statistics:



# The Topological Bridge



# The Topological Bridge

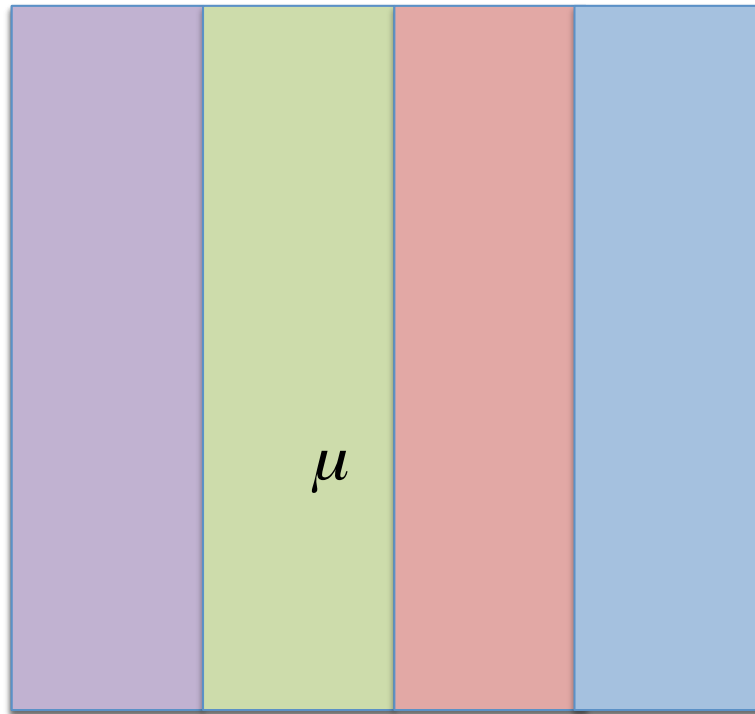
- Start with **logical** insights.
- Allow methods a small chance  $\alpha$  of error.
- Obtain corresponding **statistical** insights





# Statistical Problem

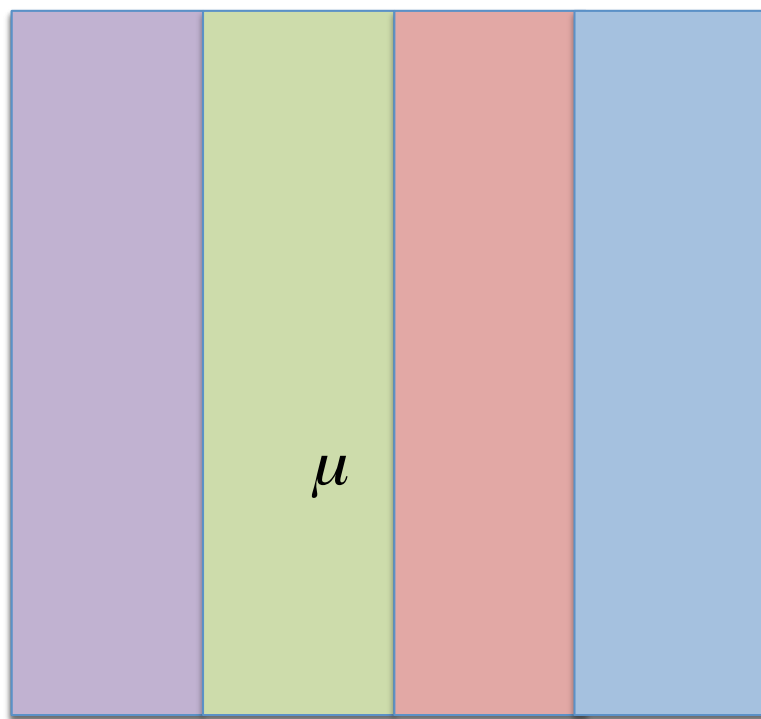
A statistical **question** partitions a set of probability measures into countably many **answers**.



# Statistical Solutions

A statistical method ( $M_n$ ) is a solution to  $\mathcal{Q}$  iff for all  $\mu$

$$\mu^n [M_n^{-1}(\mathcal{Q}(\mu))] \xrightarrow{n} 1.$$



# Recall: Ockham's Razor

**Proposition (Genin and Kelly, 2016).** The following principles are **equivalent**.

1. Infer a **simplest** relevant response in light of  $E$ .
2. Infer a **refutable** relevant response compatible with  $E$ .
3. Infer a relevant response that is **not more complex than the true answer**.

# Ockham's **Statistical** Razor

**Concern:** “consistency with  $E$ ” is **trivial** in statistics.



**Response:** the “err on the side of simplicity” version of Ockham’s razor **does not mention** consistency with  $E$ .

3. Infer a relevant response that is more complex than the true answer **with chance  $< \alpha$** .

# Ockham's **Statistical** Razor

A solution  $(M_n)$  to  $\mathcal{Q}$  satisfies **Ockham's  $\alpha$ -razor** iff

if  $A \in \mathcal{Q}$  and  $\mathcal{Q}(\mu) \triangleleft A$ , then  $\mu^n[M_n^{-1}(A)] < \alpha$ .

# Progressive Methods

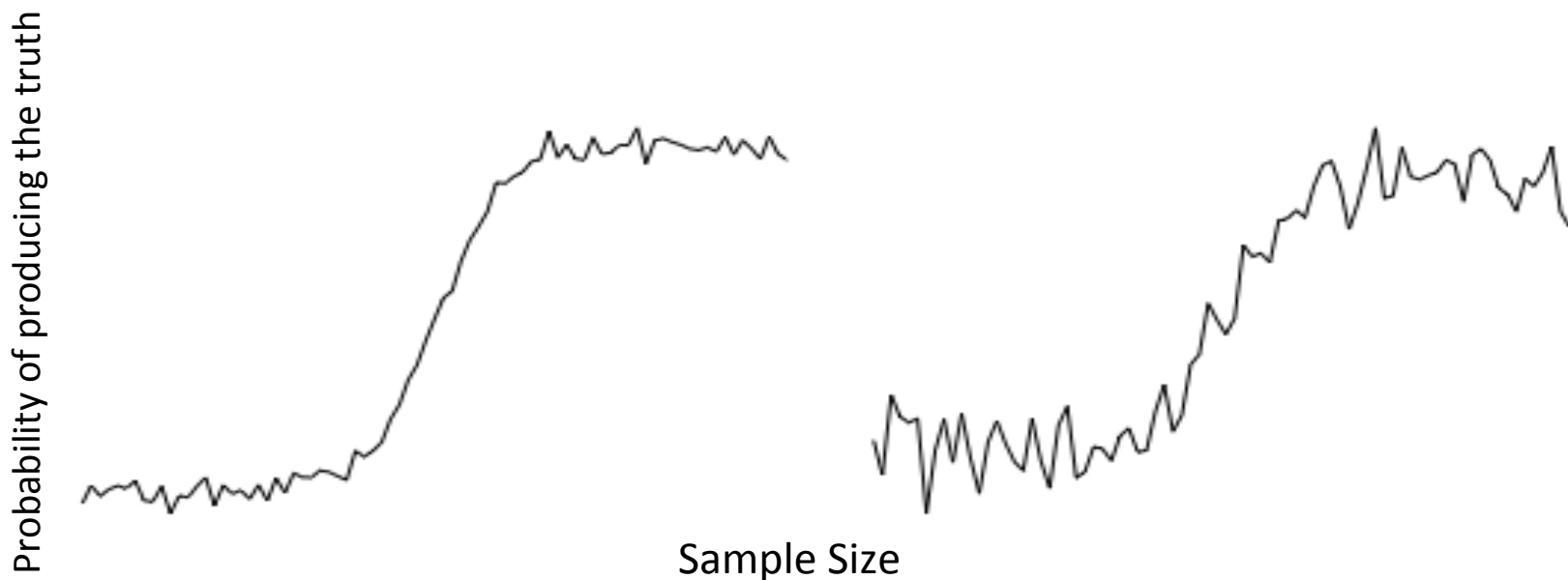
A solution  $(M_n)$  to question  $Q$  is **progressive** if the **chance** that it outputs the **true answer** is strictly **increasing** with sample size, i.e. for all  $n_1 < n_2$ :

$$\mu^{n_2} [M_{n_2}^{-1}(\mathcal{Q}(\mu))] > \mu^{n_1} [M_{n_1}^{-1}(\mathcal{Q}(\mu))].$$

# $\alpha$ -Progressive Methods

- $(M_n)$  is  $\alpha$ -progressive if the chance that it outputs the true answer **never decreases** by more than  $\alpha$ , i.e. for  $n_1 < n_2$ :

$$\mu^{n_2} [M_{n_2}^{-1}(\mathcal{Q}(\mu))] + \alpha > \mu^{n_1} [M_{n_1}^{-1}(\mathcal{Q}(\mu))].$$



# Progressive Methods

**Theorem** (Genin, 2017): If there exists an enumeration  $A_1, A_2, \dots$  of the answers to  $Q$  that agrees with the simplicity order, then there exists an  $\alpha$ -progressive method for every  $\alpha > 0$ .

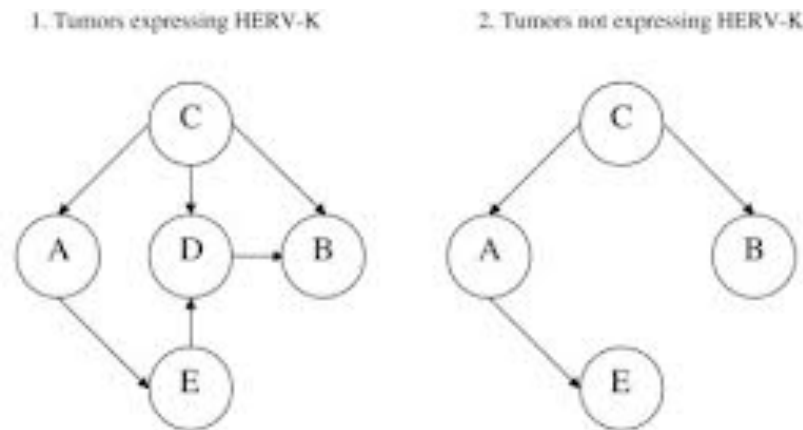


# Ockham and Progress

**Theorem** (Genin, 2017): Every  $\alpha$ -progressive solution satisfies Ockham's  $\alpha$ -razor.

# Application: Causal Inference from Non-experimental Data

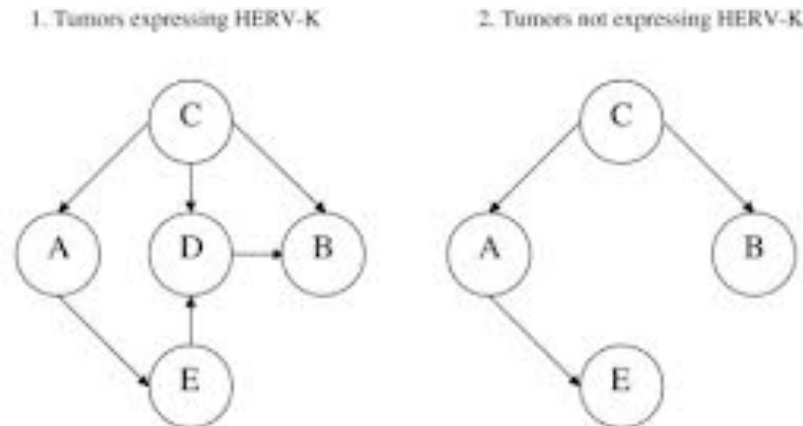
- **Causal** inference from **observational** data.
- The search is strongly guided by **Ockham's razor**.
- Previously, methods were only proven to be point-wise-consistent.



Symbols: "A" = cause (YFV); "B" = outcome (cancer); "C" = confounders (recreational solar exposure and high social class); "D" and "E" = mediators (HERV-K antigen and immune response).

# Application: Causal Inference from Non-experimental Data

**Proposition** (Genin, 2018). For the problem of inferring Markov equivalence classes, there exist  $\alpha$ -progressive solution for every  $\alpha > 0$ .



Symbols: "A" = cause (YFV); "B" = outcome (cancer); "C" = confounders (recreational solar exposure and high social class); "D" and "E" = mediators (HERV-K antigen and immune response).

Thank you!