How Inductive is Bayesian Conditioning?

Konstantin Genin
Carnegie Mellon University
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INDUCTION AND DEDUCTION

Logically Deductive Inference

Truth Preserving

- If the premises are true, the conclusion is true.

Monotonic

- Additional premises yield strictly stronger conclusions.
- Conclusions are stable in light of further premises.

Logically Deductive Inference

Truth Preserving (SYNCHRONIC)

- If the premises are true, the conclusion is true.

Monotonic (DIACHRONIC)

- Additional premises yield strictly stronger conclusions.
- Conclusions are stable in light of further premises.

Taxonomy of Inference

- Any ... **inference** in science belongs to one of **two kinds**:
  1. either it yields **certainty** in the sense that the conclusion is **necessarily true**, provided that the premises are true,
  2. or it does not.

- The first kind is ... **deductive inference** ....
- The second kind will ... be called **'inductive inference'**.

R. Carnap, *The Continuum of Inductive Methods*, 1952, p. 3 .

Frequentists Disagree

"[The] model makes certain probabilistic assumptions, from which other probabilistic implications follow deductively …

Frequentists Disagree

… Extreme p-values indicate that the data violate regularities implied by the model, or approach doing so. If these were strict violations of deterministic implications, we could just apply *modus tollens* to conclude that the model was wrong; as it is, we nonetheless have evidence and probabilities …

Frequentists Disagree

… Our view of model checking, then, is firmly in the long **hypothetico-deductive** tradition, running from Popper back through Bernard and beyond."

Gelman and Shalizi, *Philosophy and the Practice of Bayesian Statistics,* 2013.

Frequentists Disagree

“Gelman and Shalizi (2013) [discuss] the distinction between deductive reasoning (based on deducing conclusions from a hypothesis and checking whether they can be falsified, permitting data to argue against a scientific hypothesis but not directly for it) …

Frequentists Disagree

… and inductive reasoning (which permits generalization, and therefore allows data to provide direct evidence for the truth of a scientific hypothesis) …

Frequentists Disagree

It is held widely … that only deductive reasoning is appropriate for generating scientific knowledge. Usually, frequentist statistical analysis is associated with deductive reasoning and Bayesian analysis is associated with inductive reasoning."

Ionides et al., Response to the ASA's Statement on p-Values, 2017.

The Frequentist View

Disagreement focuses on synchronic issue of fallibility.

- Frequentist model falsification is deductive because it can be done with a guaranteed bound on the chance of error.

- Bayesian inference is inductive because Bayesians can become convinced of the truth of models despite having no bound on the chance of error.

Diachronic View

But what about monotonicity, and related diachronic features of deduction?

AGM BELIEF REVISION

Norms of **Qualitative** Change

Alchourrón, Gärdenfors, Makinson:

To *rationally* accommodate new evidence, one ought to (1) add only those new beliefs, and (2) remove only those old beliefs, that are *absolutely compelled* by incorporation of new information.



Norms of **Qualitative** Change

- $\mathbf{B}$ is your "belief set".
- $\mathbf{B} * E$ is the result of revising your beliefs by evidence E.

AGM Axioms

1. $\mathbf{B} * E = \mathsf{Cn}(B * E)$ Closure
2. $E \in \mathbf{B} * E$ Success
3. $\mathbf{B} * E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$ Inclusion
4. $\text{If } \mathbf{B} \nvdash \neg E, \text{ then } \mathbf{B} \subseteq \mathbf{B} * E$ Preservation
5. $E \nvdash \bot \Rightarrow \mathbf{B} * E \nvdash \bot$ Consistency
6. $\vdash X \equiv Y \Rightarrow \mathbf{B} * X = \mathbf{B} * Y.$ Extensionality

Synchronic AGM Axioms

1. $\mathbf{B} * E = \mathsf{Cn}(B * E)$ Closure
2. $E \in \mathbf{B} * E$ Success
3. $\mathbf{B} * E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$ Inclusion
4. $\text{If } \mathbf{B} \nvdash \neg E, \text{ then } \mathbf{B} \subseteq \mathbf{B} * E$ Preservation
5. $E \nvdash \bot \Rightarrow \mathbf{B} * E \nvdash \bot$ Consistency
6. $\vdash X \equiv Y \Rightarrow \mathbf{B} * X = \mathbf{B} * Y.$ Extensionality

Diachronic AGM Axioms

1. $\mathbf{B} * E = \mathrm{Cn}(B * E)$ Closure
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AGM3: Inclusion

$$\mathbf{B} * E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$$

Believe no more than the deductive closure of your old beliefs + your evidence.

AGM3: Inclusion

$$\mathbf{B} * E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$$

Slogan: No induction, without refutation!

Genin and Kelly, *Learning, Theory Choice and Belief Revision*, forthcoming.

AGM4: Preservation

$$\text{If } \mathbf{B} \nvdash \neg E, \text{ then } \mathbf{B} \subseteq \mathbf{B} * E.$$

Believe no less than the deductive closure of your old beliefs + your evidence (so long as the evidence is consistent with your prior beliefs).

AGM4: Preservation

$$\text{If } \mathbf{B} \nvdash \neg E, \text{ then } \mathbf{B} \subseteq \mathbf{B} * E.$$

Slogan: No retraction, without refutation!

Genin and Kelly, *Learning, Theory Choice and Belief Revision*, forthcoming.

AGM 3+4

$$\text{If } \mathbf{B} \nvdash \neg E, \text{ then } \mathbf{B} * E = \mathsf{Cn}(B \cup \{E\}).$$

Proceed deductively, unless your beliefs are refuted.

(I am assuming Success and Closure.)

THE INCLUSION PRINCIPLE

AGM3: Inclusion

$$\mathbf{B} * E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$$

Suppose $\mathbf{B} * E \vdash H$.

Then, by Inclusion, $\mathbf{B}, E \vdash H$.

By the Deduction Theorem, $\mathbf{B} \vdash E \supset H$.

AGM3: Inclusion

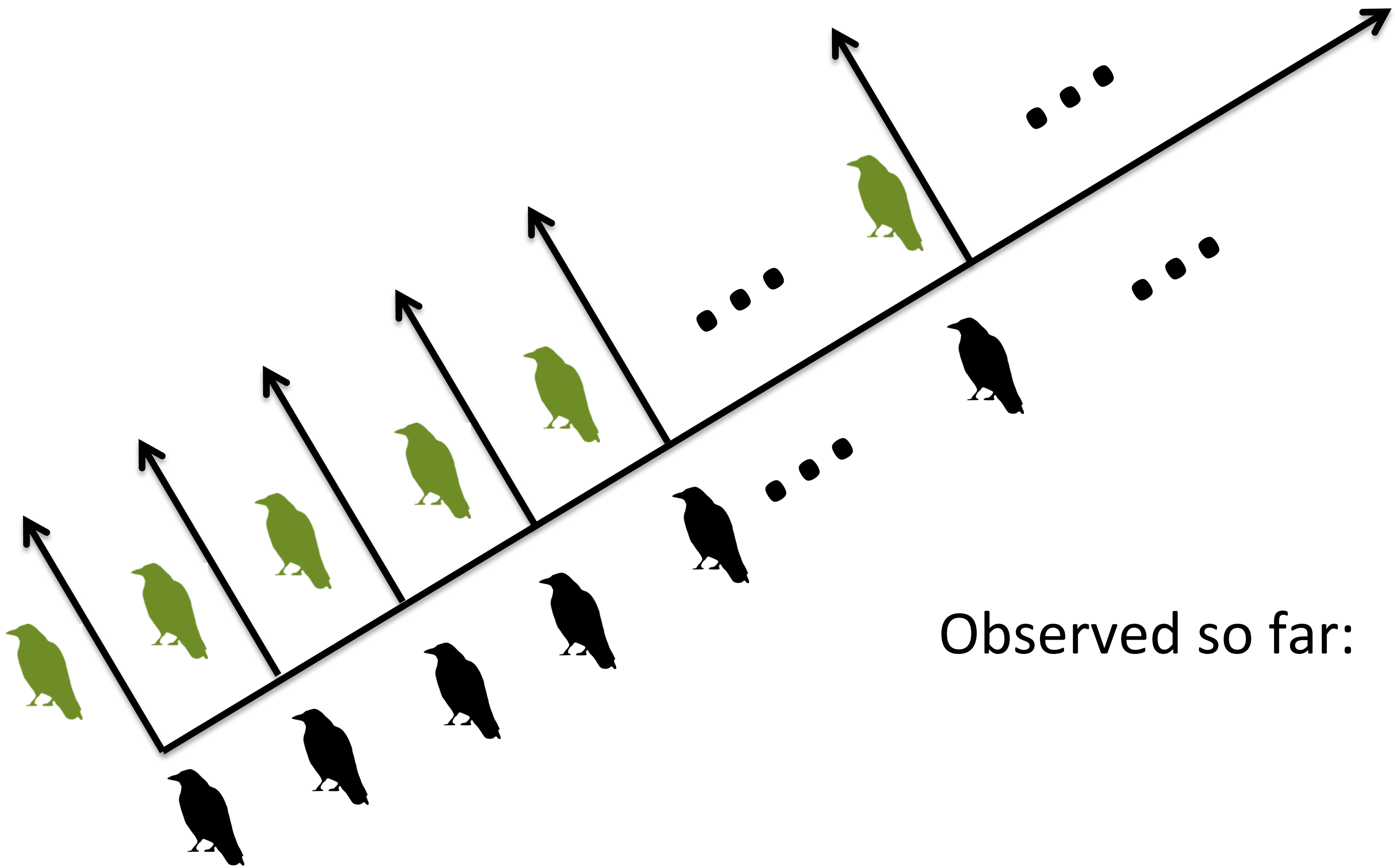
“Inductive generalizations … accompany belief expansions by new observations, in science as well as in common sense cognitions. After observing several instances of a ‘constant conjunction’, humans almost automatically form the corresponding inductive generalization; and after performing a new experimental result sufficiently many times, experimental scientists proclaim the discovery of a new empirical law …

AGM3: Inclusion

… AGM-type expansion is not at all creative but merely additive: it simply adds the new information and forms the deductive closure, but never generates new (non-logically entailed) hypotheses.

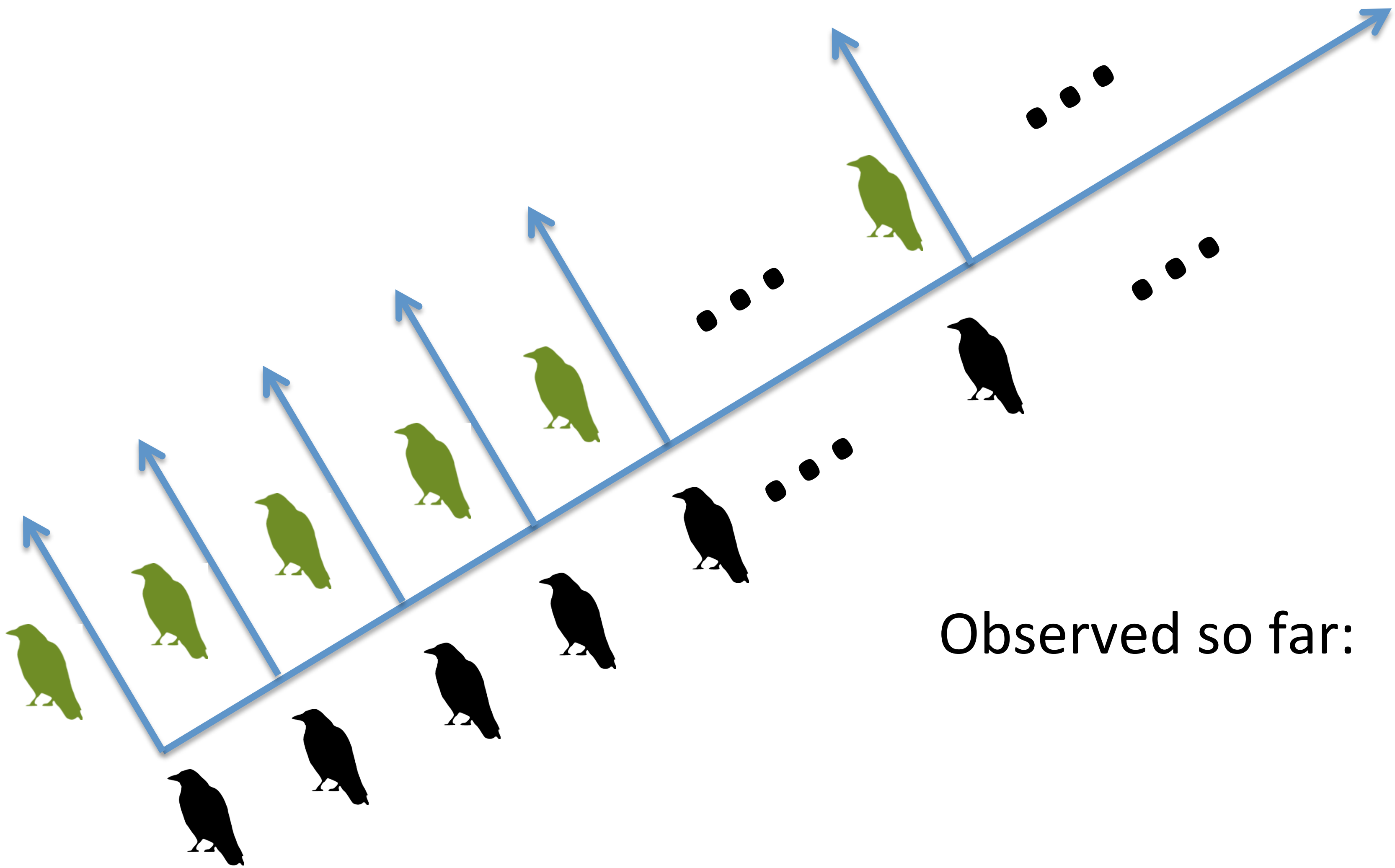
G. Schurz, *Abductive Belief Revision in Science*, 2011.

Conjecture:

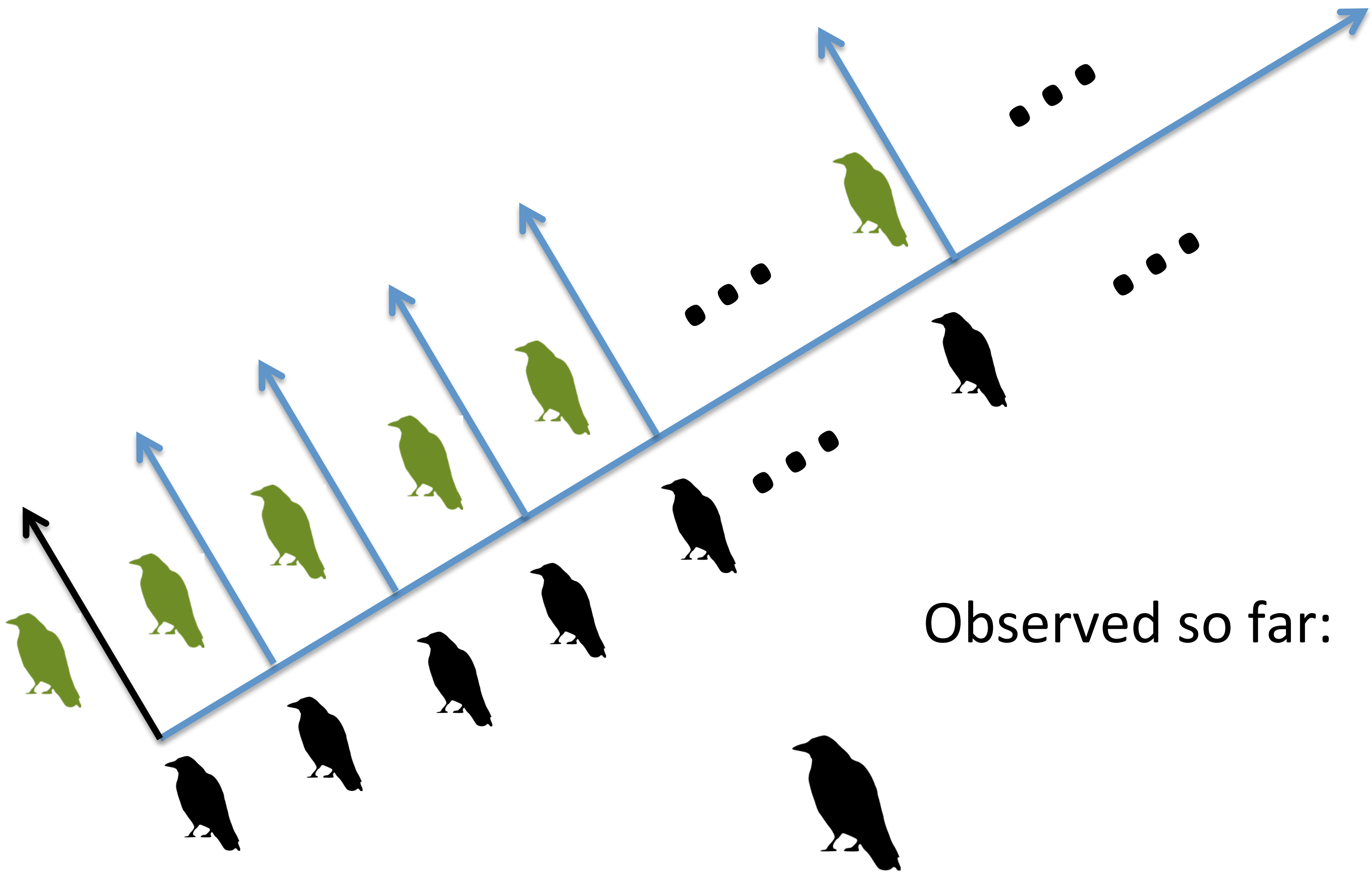


Observed so far:

Conjecture: Who knows?

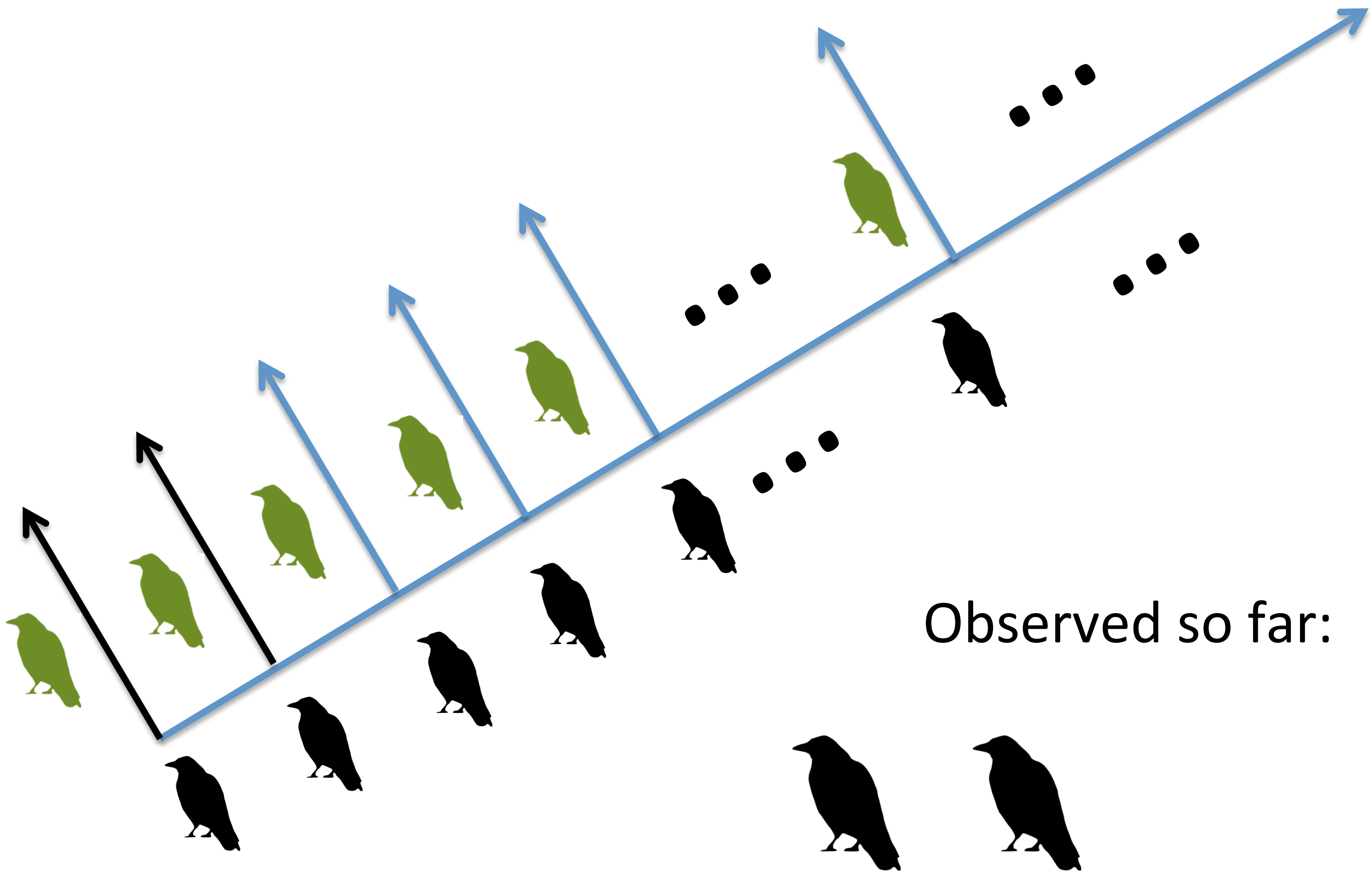


Conjecture: All I can say is that the first raven is black.

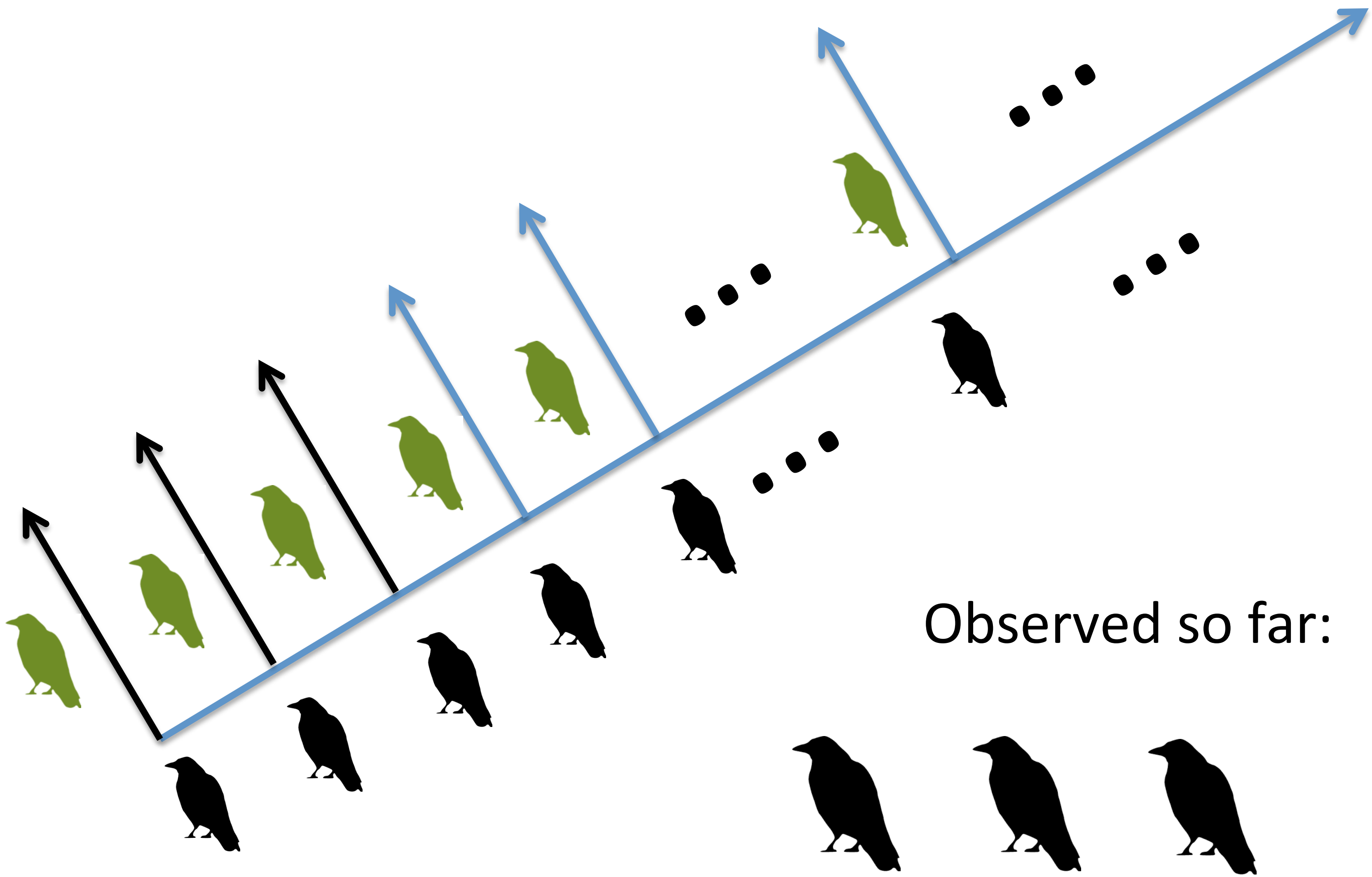


Observed so far:

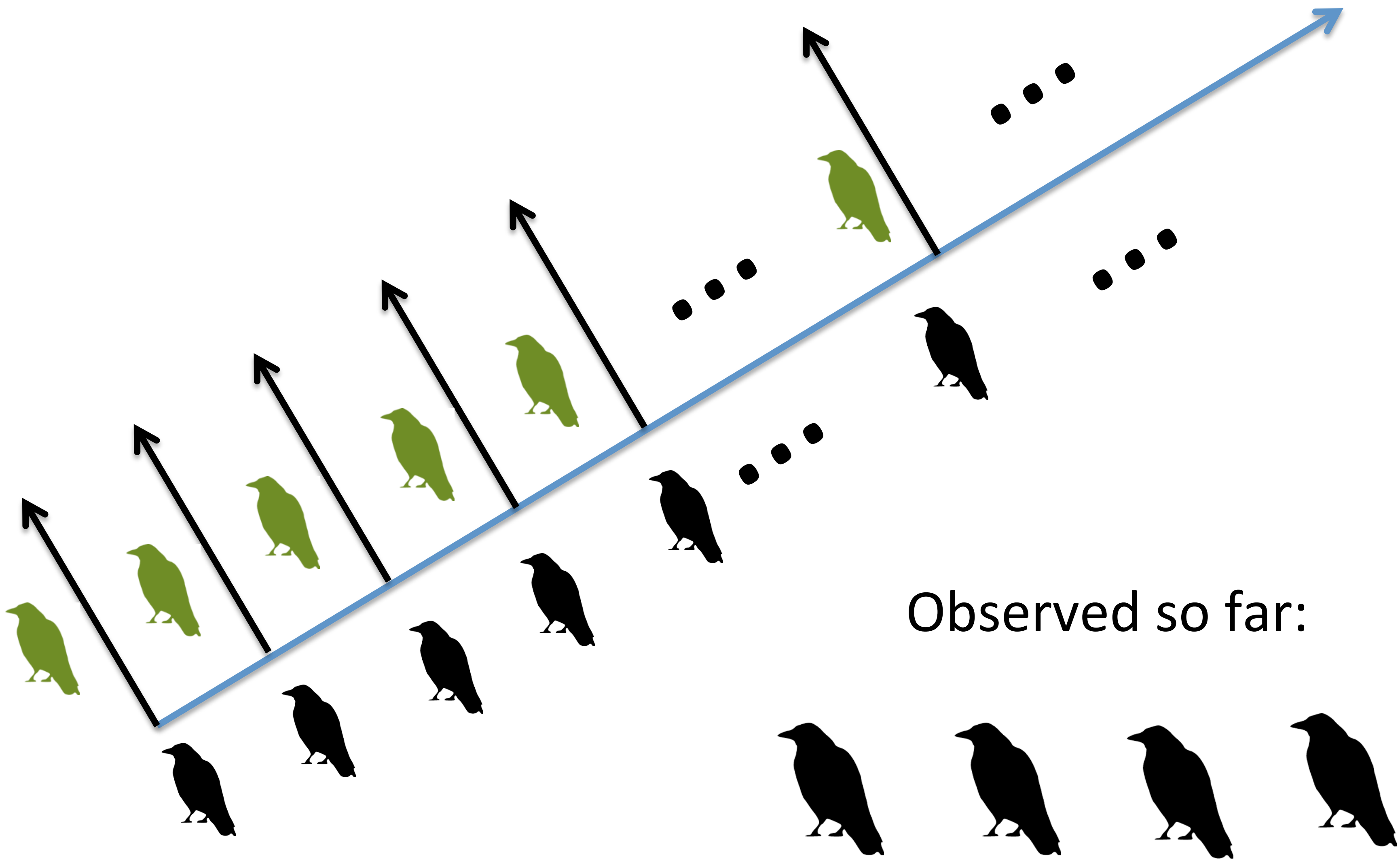
Conjecture: All I can say is that the first 2 ravens are black.



Conjecture: All I can say is that the first 3 ravens are black.



Conjecture: Fine, all ravens are black!



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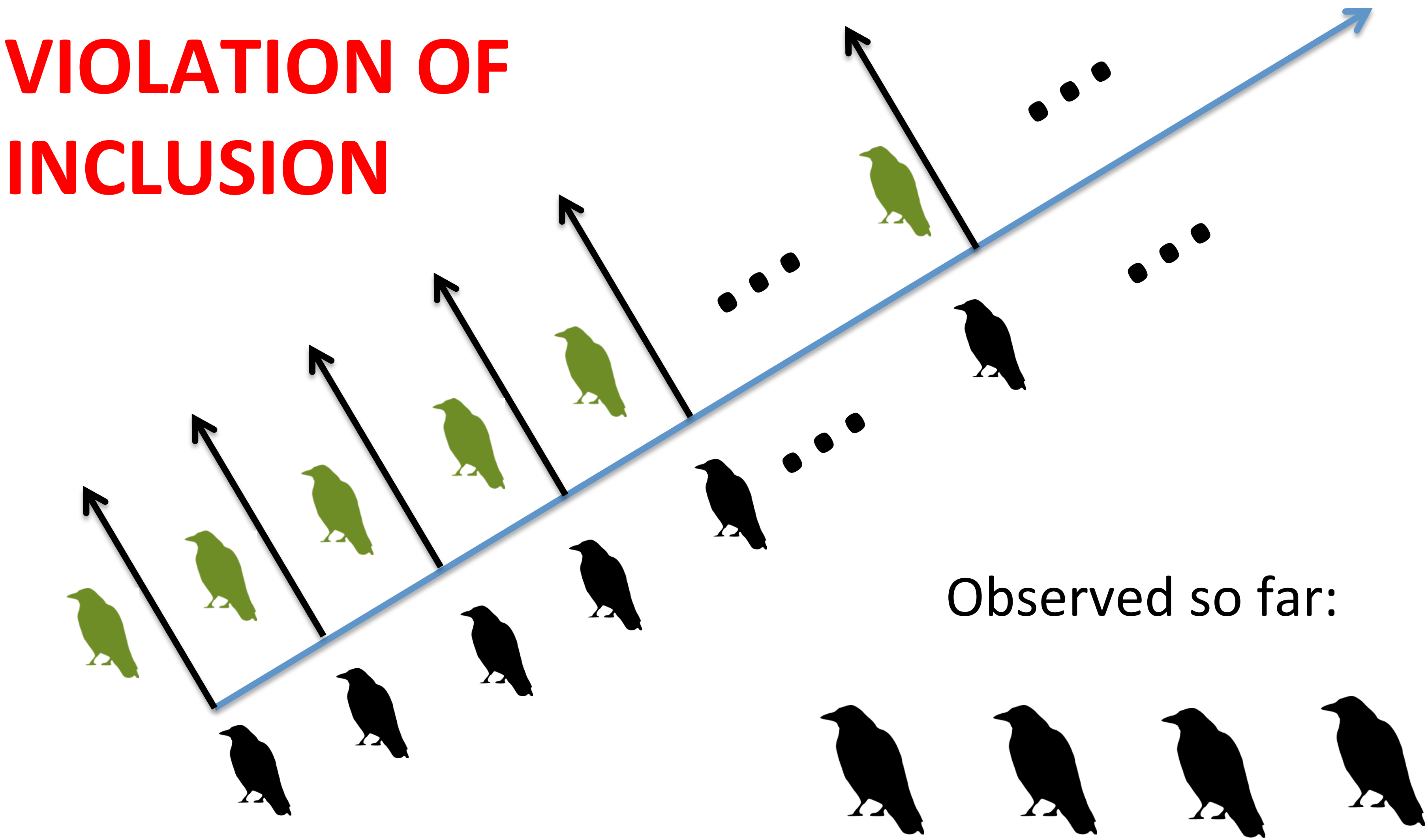
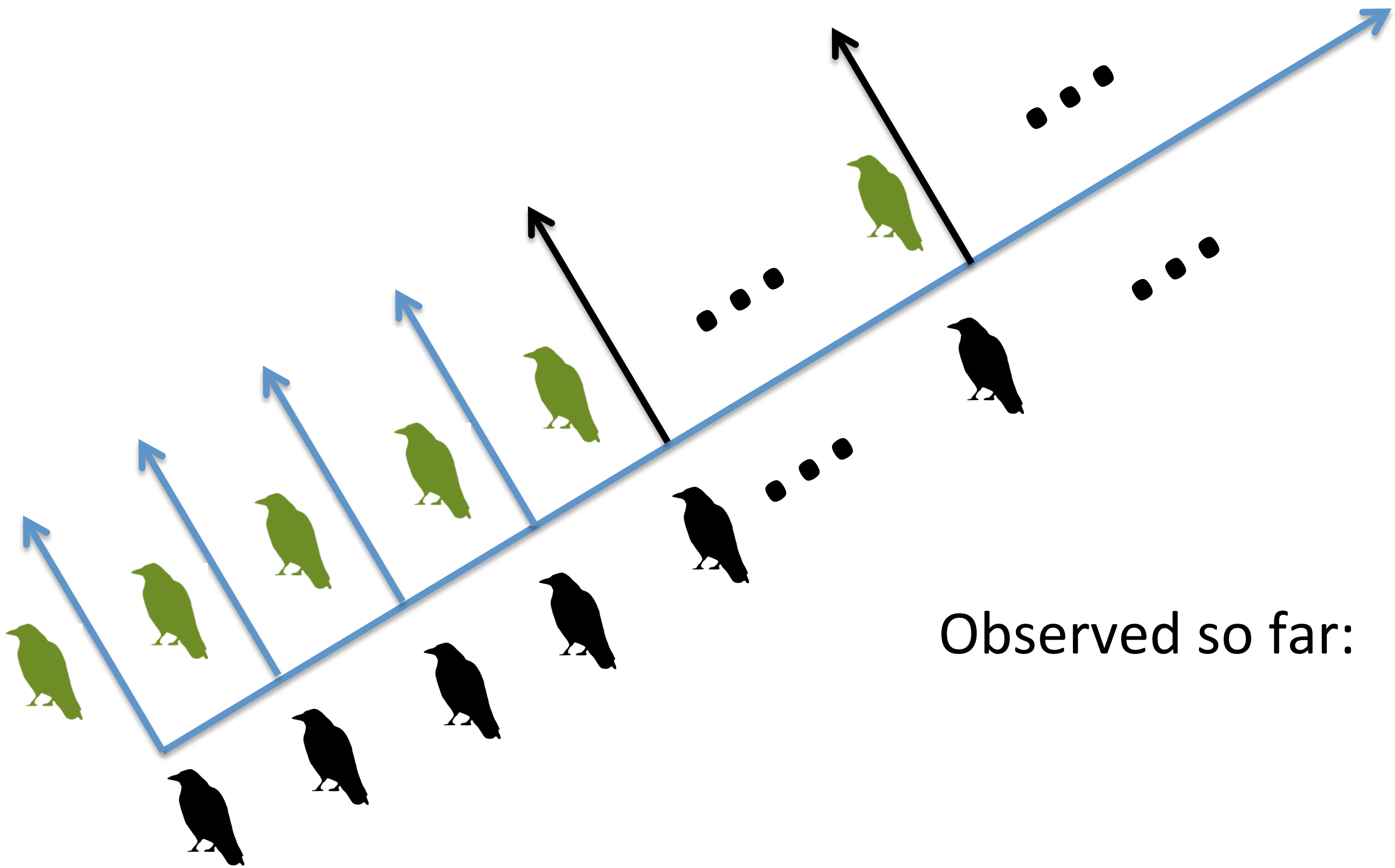
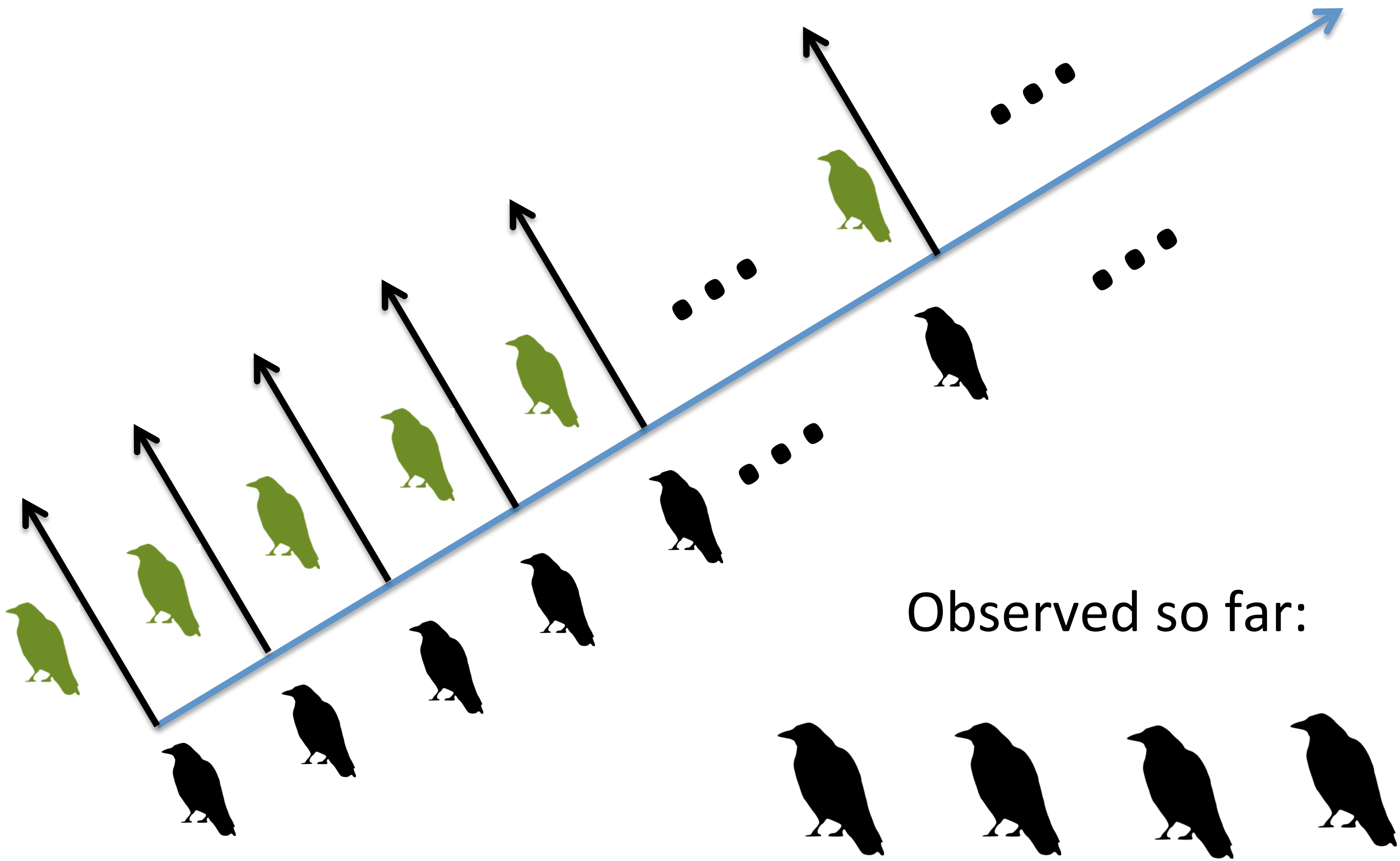


Conjecture: Either all ravens are black, or the first non-black raven appears among the first 4.



Observed so far:

Conjecture: All ravens are black!



Observed so far:

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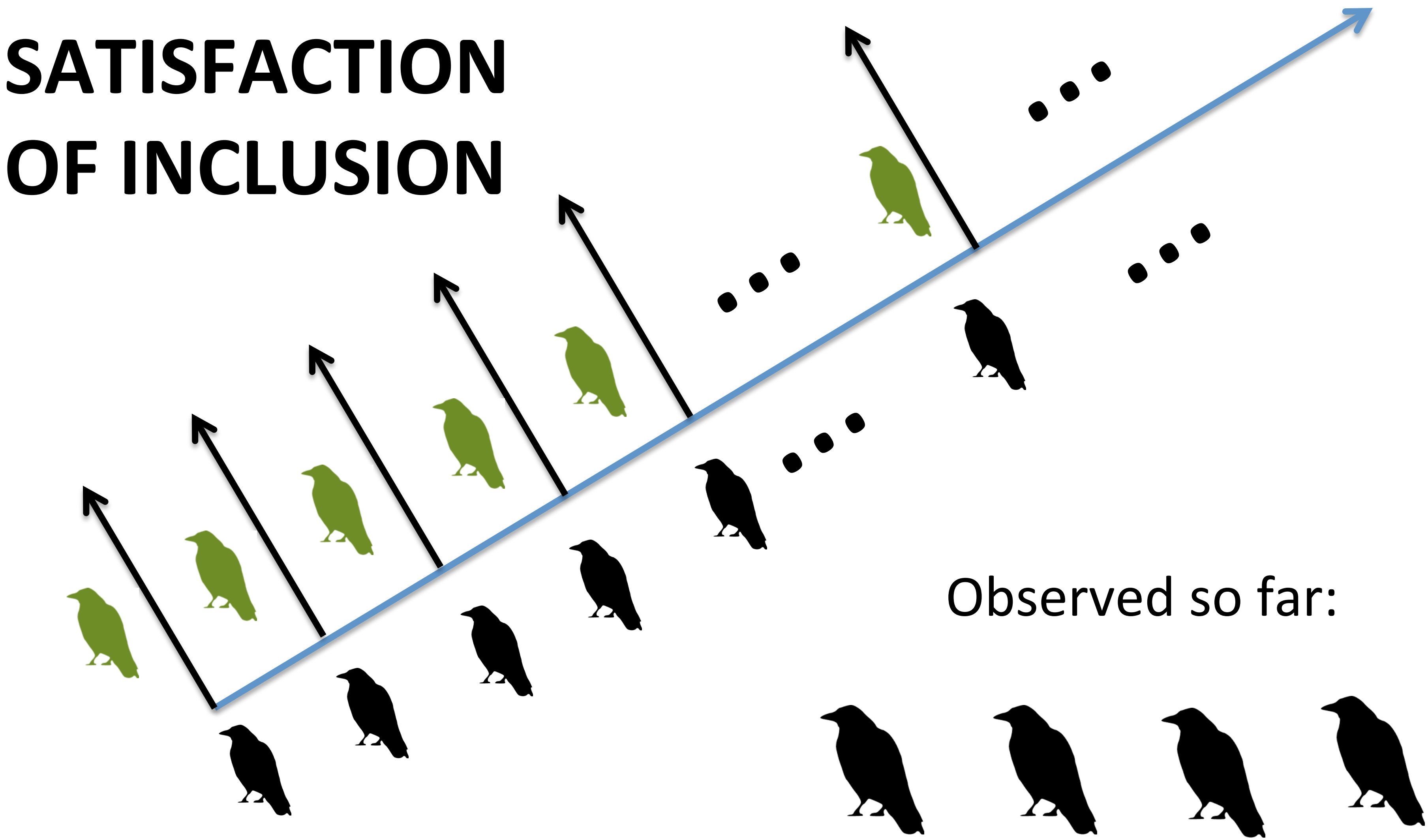


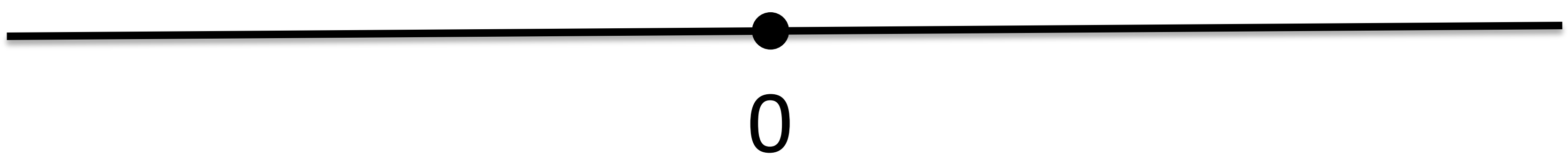
AGM3: Inclusion

Suppose that 1) Emily satisfies Inclusion and 2) after observing that the first *n* ravens are black, she believes that all ravens are black.

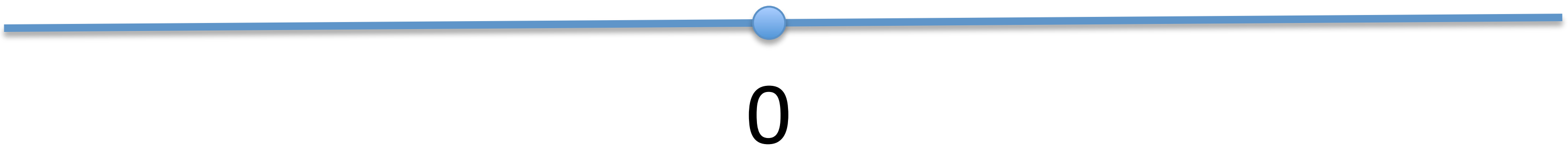
Therefore, *ex ante*, Emily believes:

1) if the first *n* ravens are all black, then all ravens are black;
2) if some ravens are not black, then the first non-black raven appears among the first *n* observed ravens;
3) either all ravens are black or the first non-black raven appears among the first *n.*

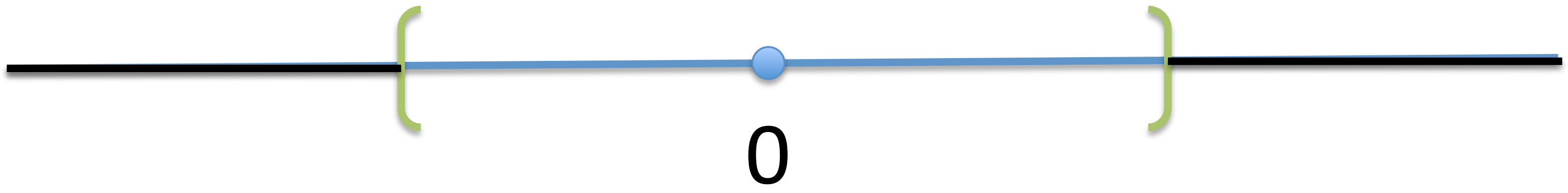
Conjecture:



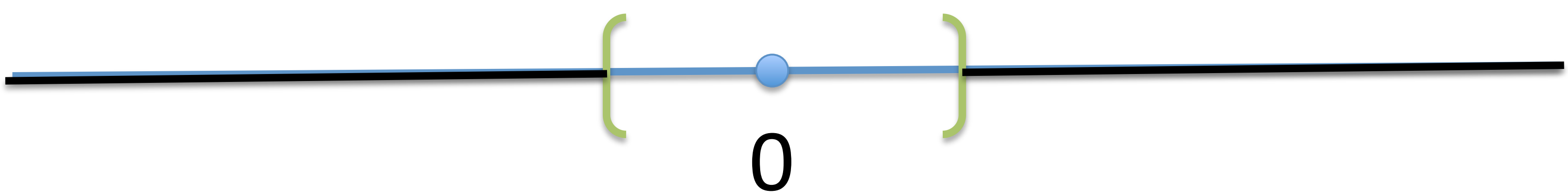
Conjecture: The luminiferous aether may have any velocity or it may not exist at all.



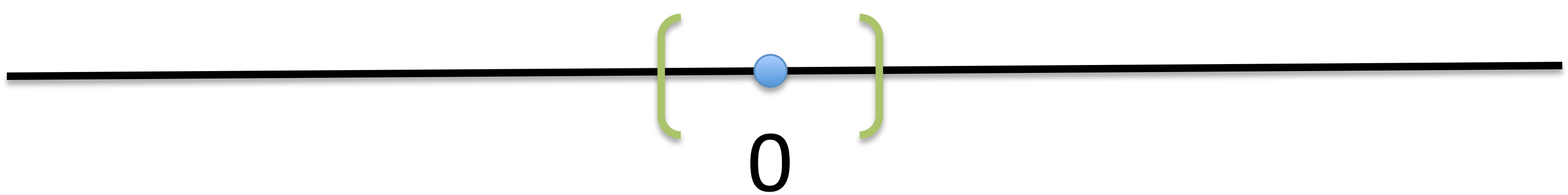
Conjecture: If the aether wind exists, it moves kind of slow.



Conjecture: If the aether wind exists, it moves *very* slow.

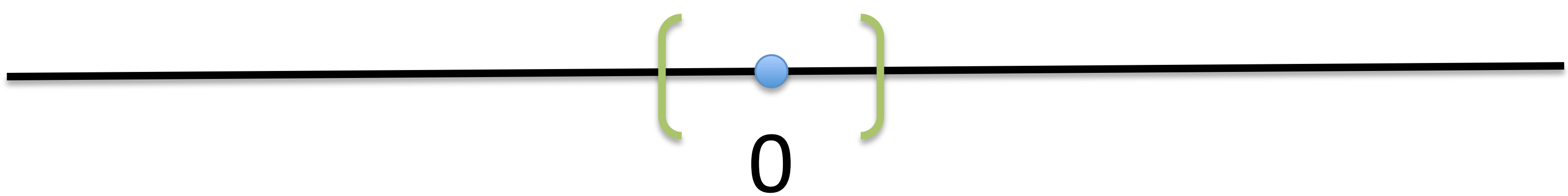


Conjecture: Fine, there is no aether wind!

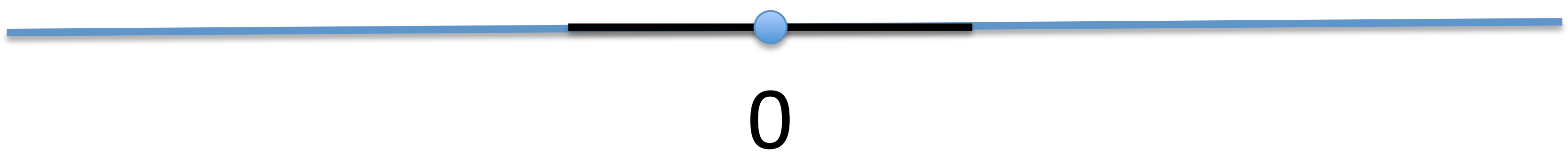


Conjecture: Fine, there is no aether wind!

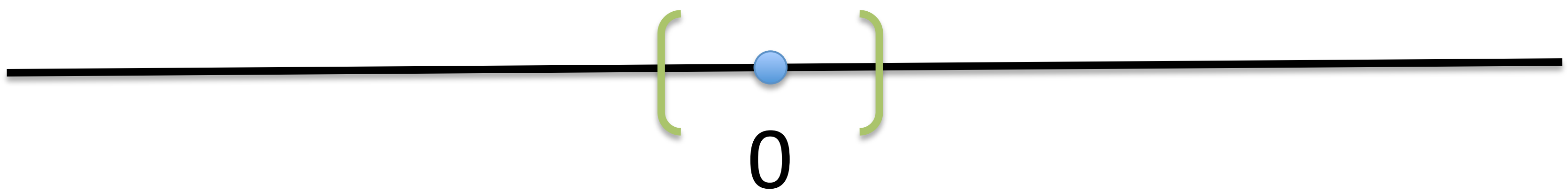
VIOLATION OF INCLUSION



Conjecture: If the aether wind exists, it is pretty fast.

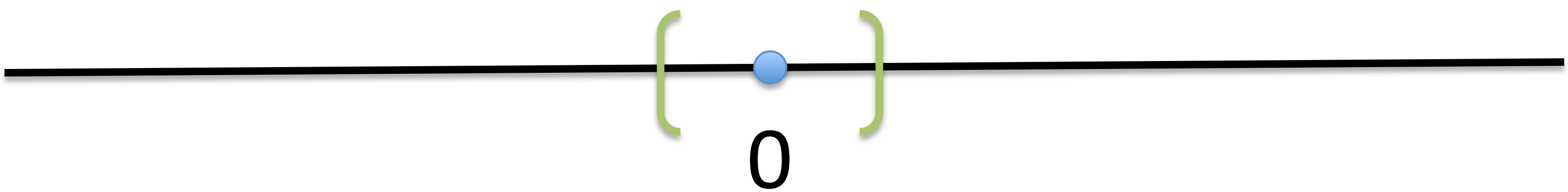


Conjecture: There is no aether wind!



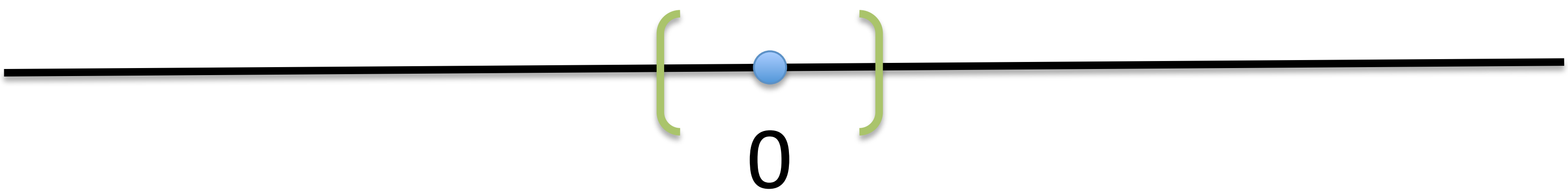
Conjecture: There is no aether wind!

SATISFACTION OF INCLUSION



Conjecture: There is no aether wind!

SATISFACTION OF INCLUSION



Inductive Dogmatism

An inductive **dogmatist** w.r.t H is an agent that is certain *a priori* that the problem of induction never arises.

Inductive Dogmatism

Theorem. Any agent that learns H and satisfies Inclusion is an inductive dogmatist w.r.t H.

Genin and Kelly, *Learning, Theory Choice and Belief Revision*, forthcoming.

BAYESIAN CONDITIONING

Bayesian Conditioning and Inclusion

Fact.

$$P(H|E) \leq P(E \supset H)$$

Bayesian Conditioning and Inclusion

Fact.

$$P(H|E) \leq P(E \supset H)$$

Conditioning satisfies "quantitative" Inclusion.

Lockean Thesis

$$\text{For some } t \in (1/2, 1] :$$

$$X \in \mathbf{B}_P \text{ iff } P(X) \geq t.$$

Lockean Update

For some $t \in (1/2, 1]$:

$X \in \mathbf{B}_P * E$ iff $P(X|E) \geq t$.

Lockean Update and Inclusion

Theorem. Lockean Update satisfies Inclusion.

Shear, Fitelson, Weisberg. Two Approaches to Belief Revision. *2017.*

Proof.

Suppose $t \leq P(H|E)$.

By the fact, $P(H|E) \leq P(E \supset H)$.

So $H \in \mathsf{Con}(\mathbf{B}_P \cup \{E\})$.

Jeffrey-Lockean Update and Inclusion

Fitelson's Conjecture: If we replaced extremal conditioning with Jeffrey conditioning, the resulting agent would not necessarily satisfy Inclusion.

Jeffrey-Lockean Update and Inclusion

Fitelson's Conjecture: If we replaced extremal conditioning with Jeffrey conditioning, the resulting agent would not satisfy inclusion.

The conjecture is False ☹

Jeffrey Conditioning

$$\begin{aligned} P_{new}(H) =& P_{old}(H|E)P_{new}(E) + \\ & P_{old}(H|\neg E)(1 - P_{new}(E)) \end{aligned}$$

Jeffrey Conditioning

$$\begin{aligned} P_{new}(H) = & P_{old}(H|E)P_{new}(E) + \\ & P_{old}(H|\neg E)(1 - P_{new}(E)) \end{aligned}$$

Meant to capture the effect of updating on uncertain evidence.

Jeffery Conditioning and Inclusion

Theorem. (Genin) If $0 < P_{old}(E) \leq P_{new}(E)$, then $P_{new}(H) \leq P_{old}(E \supset H)$.

Jeffery Conditioning and Inclusion

Theorem. (Genin) If $0 < P_{old}(E) \leq P_{new}(E)$, then $P_{new}(H) \leq P_{old}(E \supset H)$.

So Jeffery conditioning also satisfies "quantitative" Inclusion.

Jeffery Conditioning and Inclusion

Theorem. (Genin) If $0 < P_{old}(E) \leq P_{new}(E)$**,** then $P_{new}(H) \leq P_{old}(E \supset H)$.

And a Lockean agent updating by Jeffrey conditionalization satisfies Inclusion.

How Inductive is Bayesian Conditioning?

"… Konstantin Genin and Kevin Kelly point out [that] on the face of it, this fact suggests that Lockeanism is committed to deductivism about inductive inference. … if any proposition newly learned by a Lockean could have been learned by deduction using the new evidence and old beliefs, then it may seem that the inductive apparatus plays an inessential role in learning. However, we suspect that this inference is a bit too quick … acquiring new evidence can undermine an agent's old beliefs and, thus, render them unfit for use in such an inference."

Shear, Fitelson, Weisberg. Two Approaches to Belief Revision. *2017.*

Bayesian Conditioning and Preservation

Recall that Preservation requires that:

$$\text{If } \mathbf{B} \nvdash \neg E, \text{ then } \mathbf{B} \subseteq \mathbf{B} * E.$$

Bayesian Conditioning and Preservation

Let

- a := Emily is in Amsterdam;
- b := Emily is in Berlin;
- n := Emily is in New York.

Suppose $P(b) = .8$, $P(a)$=$.11$, and $P(n)$=$.09$. If my Lockean threshold is .9, Then:

$$\mathbf{B}_P = \{b \vee a, a \vee b \vee n\}.$$

So I believe Emily is in Europe. Suppose I learn that Emily is not in Berlin. Now, all I believe is that Emily is either in New York or Amsterdam. So I no longer believe she is in Europe! Preservation is violated.

Bayesian Conditioning and Preservation

Theorem. (Shear, Fitelson, Weisberg) If a Lockean agent satisfies the synchronic requirements of AGM, and her threshold is in the interval $[.5, \phi^{-1})$, then she satisfies Preservation.

Shear, Fitelson, Weisberg. Two Approaches to Belief Revision. *2017.* Theorem 1.

$$\phi^{-1} \approx .618.$$

Bayesian Conditioning and Preservation

Lemma. (Genin)

$$\text{If } P(E) < \phi^{-1} \text{ and } P(H|E) < \phi^{-1},$$

$$\text{then } P(E \supset \neg H) > \phi^{-1}.$$

Bayesian Conditioning and Preservation

Lemma. (Genin)

$$\text{If } P(E) < \phi^{-1} \text{ and } P(H|E) < \phi^{-1},$$

$$\text{then } P(E \supset \neg H) > \phi^{-1}.$$

So the extent of non-deductive undermining is qualified.

Thank you!