A Topological Theory of Empirical Simplicity

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Rhetoric $\neq$ Reliability

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- An entirely different question is the problem of the **validity** of our or any other proposed system of inductive logic, and thereby of the customary methods of inductive thinking. This is the genuinely philosophical problem of induction.
- The construction of a systematic inductive logic is an important step towards the solution of the problem, but still only a preliminary step. (pp. 95-6).

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2. the fact that the truth of the predictions reached by induction cannot be guaranteed does not preclude a justification in a weaker sense;
3. . . . [he] shows that his rule of induction R leads to **success** in the following sense: R yields in the long run an approximate estimate of the relative frequency in the whole of any given property (p. 96).

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- However, Reichenbach's rule and the other ... rules mentioned yield different numerical values ... although those values converge for an increasing sample toward the same limit.
- Therefore we need a more general and stronger method for examining and comparing any two given rules of induction ... (p. 97).

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- One has only the sense that unless the world is perversely complex, the strategy will help us to locate false hypotheses and separate them from true ones. A sense is not an argument (p. 377).

Rhetoric $\neq$ Reliability

What Scientific Rhetoric Amounts to

- uniformity
- testability
- parsimony
- unity
- explanatory power
- accuracy

These are all aspects of Ockham's razor.

Ockham's Razor

Three Fundamental Questions

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I. What is empirical simplicity?

II. Given simplicity, what is Ockham's razor?

III. How does Ockham's razor help you find the truth better than alternative methods?

Section 1

Empirical Simplicity

I. What is Empirical Simplicity?

Some Standard Accounts

- uniformity of nature (grue/bleen?)
- entrenchment (new vocabulary?)
- compressibility (which computer language?)
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Our Account

- Empirical simplicity is iterated problems of induction in the empirical problem one faces.
- Thus, it depends on the question one is asking and the information one could receive in the future.

I. What is Empirical Simplicity?

Empirical Problem

$\mathfrak{P} = (W, \mathcal{I}, \mathcal{Q})$, where:

- W = a set of possibilities.
- $\mathcal{I}$ = a countable set of information states satisfying:
  1. $\mathcal{I}$ covers W;
  2. each pair of true information states is entailed by a true information state.
- $\mathcal{Q}$ is a question that partitions W into countably many potential answers (hypotheses, theories, or research programs).

I. What is Empirical Simplicity?

Information Topology

$\mathcal{I}^*$ = the closure of $\mathcal{I}$ under arbitrary union (disjunction).

- Open propositions are verifiable.
- Closed propositions refutable.
- A° = "A will be verified".
- $\overline{A}$ = "A will never be refuted".

Kelly (1996), Luo and Schulte (2006), deBrecht and Yamamoto (2008), Baltag, Smets, and Gierasimczuk (2014).

I. What is Empirical Simplicity?

The Specialization Pre-order

$A \preceq B$ iff $A \subseteq \overline{B}$.

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- A entails that B will never be refuted (the problem of induction).
- A is as falsifiable as B (Popper).

I. What is Empirical Simplicity?

Two Wrinkles Concerning $\preceq$

1. Cycles.
2. Non-rigidity in light of new information.

I. What is Empirical Simplicity?

Solution

- Substitute a better question $\mathcal{S}$ for the original question $\mathcal{Q}$.
- Call $\mathcal{S}$ a simplicity concept for $\mathfrak{P}$ and call answers to $\mathcal{S}$ simplicity degrees.
- $\mathcal{S}$ is related to $\mathfrak{P}$ by three axioms.

I. What is Empirical Simplicity?

Axiom 1. Local Closure

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- A implies that A will be refutable.
- "If B is verified, then conclude A until C is refuted".
- **Proposition**. If each simplicity degree in $\mathcal{S}$ is locally closed, then $(\mathcal{S}, \preceq)$ is a partial order.

I. What is Empirical Simplicity?

Axiom 2. Homogeneity

- $\mathcal{S}$ is homogeneous for $\mathfrak{P}$ iff

$$A \cap \overline{B} \neq \varnothing \quad \Rightarrow \quad A \preceq B$$

  for all $A, B \in \mathcal{S}$.

- **Proposition**. The simplicity relation $\preceq$ is stable under restriction by new information iff $\mathcal{S}$ is homogeneous for $\mathfrak{P}$.

I. What is Empirical Simplicity?

Axiom 3. Makes the Original Question Decidable

- $\mathcal{S}$ decides $\mathfrak{P}$ iff each answer to $\mathcal{Q}$ is open (verifiable) in the information topology restricted to an arbitrary element of $\mathcal{S}$.

Section 2

What is Ockham's Razor?

II. What is Ockham's Razor?

Ockham's Vertical Razor

- Your belief state should be closed downward in $\preceq$.

Ockham's Horizontal Razor

- Your belief state should be co-initial in $\preceq$.

Section 3

How Does Ockham's Razor Help You Find the Truth?

III. How Does Ockham's Razor Help You Find the Truth?

Bayesian answer

- Simpler worlds are more probable, so Ockham's Razor is probably right.
- Bayes converges to the truth in the long run, but so do infinitely many other methods.

Frequentist answer

- Estimates based on simpler models have lower variance.
- Doesn't converge to the true model at all (AIC).

III. How Does Ockham's Razor Help You Find the Truth?

Problem Solution in the Limit

- A method λ returns propositions in response to information states.
- Method λ solves $\mathfrak{P}$ in the limit iff
  - for each $w \in W$
  - there exists information state E true in w such that
  - for each information state F true in w that entails E:

$$\varnothing \neq \lambda(F) \subseteq \text{the simplicity degree in } \mathcal{S} \text{ that is true in } w.$$

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- But it is impossible to produce the inductive truth now.
- So is there an intermediate concept of success that is inductively feasible and that mandates Ockham's razor now?

III. How Does Ockham's Razor Help You Find the Truth?

Ancient Hint (Katha Upanishad, Müller Translation)

- Fools dwelling in darkness, wise in their own conceit, and puffed up with vain knowledge
  - go round and round
  - staggering to and fro,

  like blind men led by the blind.

III. How Does Ockham's Razor Help You Find the Truth?

Staggering To and Fro = Doxastic Reversals

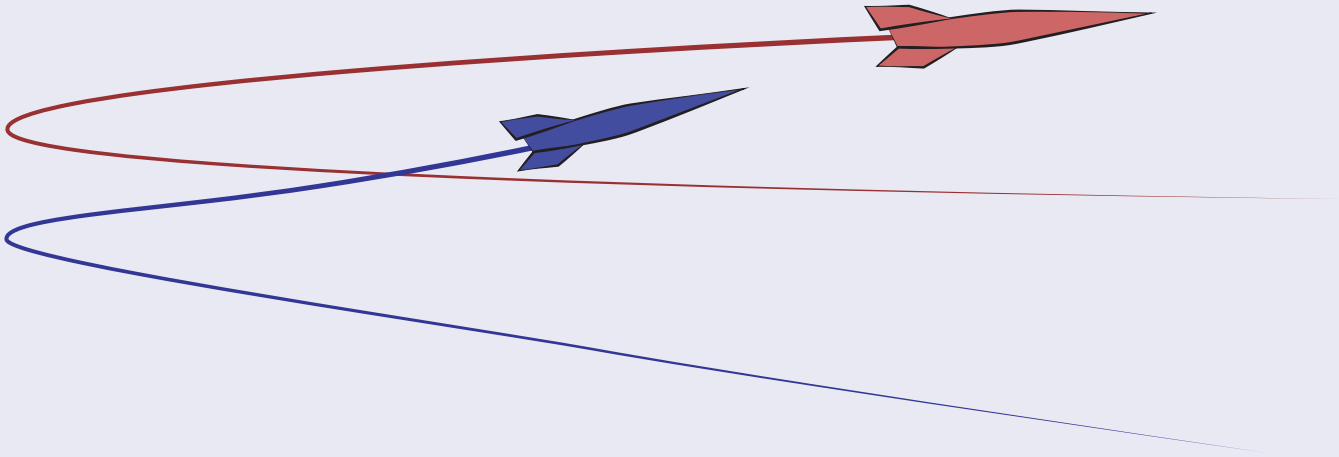
1. Believe A;
2. believe B inconsistent with A.

Going Round and Round = Doxastic Cycles

1. Believe A;
2. believe B inconsistent with A;
3. believe C that entails A.

III. How Does Ockham's Razor Help You Find the Truth?

Conducive Pursuit



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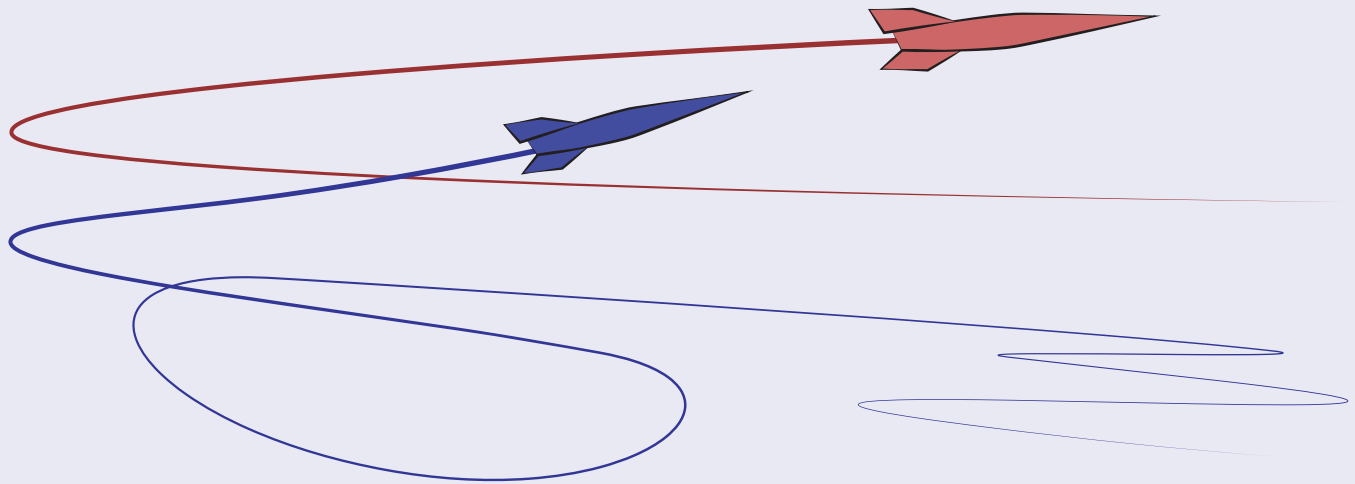
Non-conducive Pursuit



III. How Does Ockham's Razor Help You Find the Truth?

Direct Comparison of Reversal and Cycle Sequences

- $\sigma \leq \tau$ iff there exists sub-sequence τ' of τ such that $\tau'_i \subseteq \sigma_i$, for all $i \leq$ length of σ.

III. How Does Ockham's Razor Help You Find the Truth?

Worst-case Comparisons over Simplicity Degrees

- Let $E \in \mathcal{I}$.
- $\lambda' \leq_E^{\text{rev}} \lambda$ iff for each reversal sequence σ generated by λ, in world $w \in E$, there exists reversal sequence τ produced by λ' in world $v \in C \cap E$ such that $\sigma \leq \tau$.
- Similarly for cycles.

III. How Does Ockham's Razor Help You Find the Truth?

Optimality

- λ is reversal optimal in $\mathfrak{S}$ iff
  1. λ solves $\mathfrak{S}$ in the limit;
  2. $\lambda' \leq_E^{\text{rev}} \lambda$, for all $E \in \mathcal{I}$ and for all λ' that solve $\mathfrak{S}$ in the limit.
- Similarly for cycle-optimality.

III. How Does Ockham's Razor Help You Find the Truth?

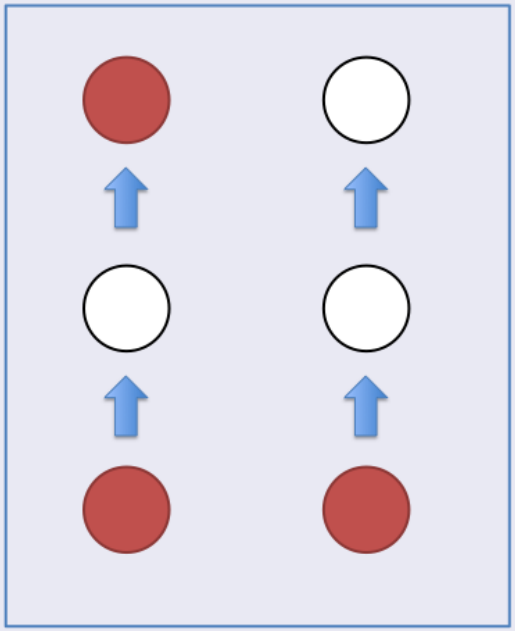
Sub-optimality

- λ is reversal sub-optimal in $\mathfrak{S}$ iff
  1. λ does not solve $\mathfrak{S}$ in the limit or
  2. $\lambda <_E^{\text{rev}} \lambda'$, for some $E \in \mathcal{I}$ and for some λ' that solves $\mathfrak{S}$ in the limit.
- Similarly for cycle-optimality.

Justification of Ockham's Razor

Vertical Razor

- Necessary for cycle optimality.
- Necessary for avoidance of cycle sub-optimality.



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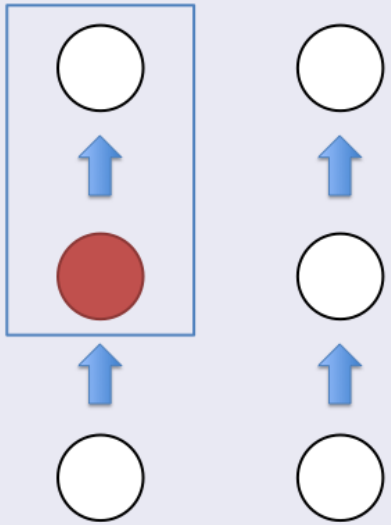

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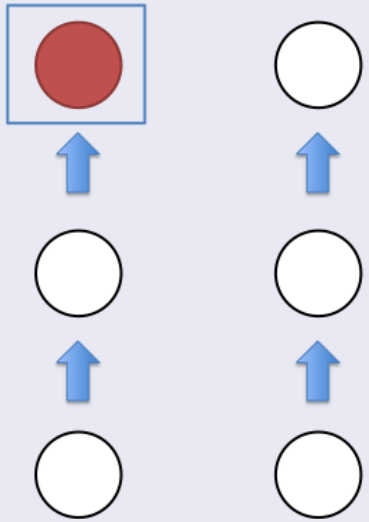

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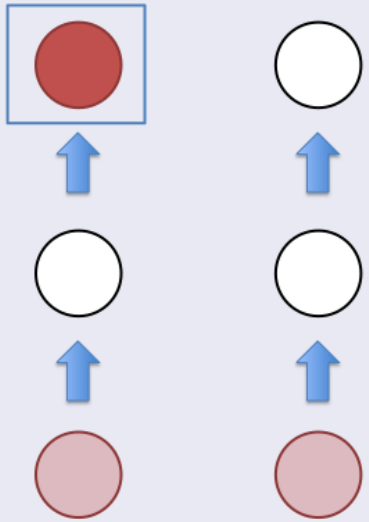

Justification of Ockham's Razor

Horizontal Razor

Assume that lower bounds on empirical complexity are verifiable. Then:

- Necessary and sufficient for reversal optimality.
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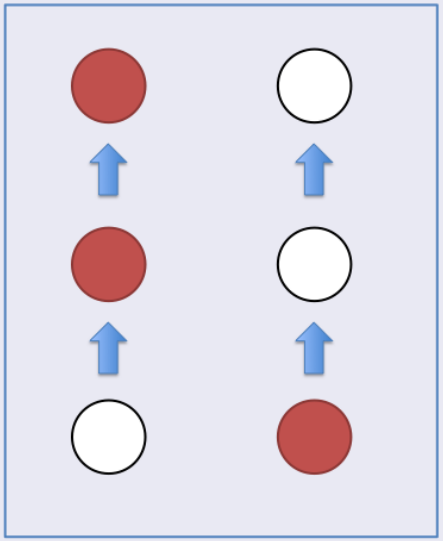

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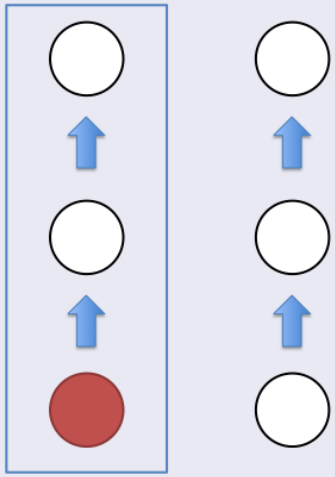

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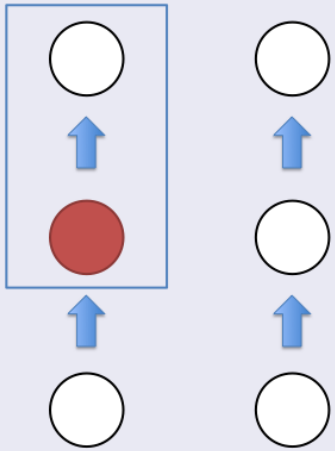

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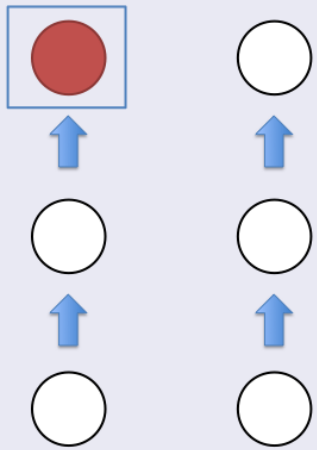

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